Deutsche Telekom Chair of Communication Networks
Technische Universität Dresden

Analog Inter Flow Network Coding

Dongho You // Summer Semester 2020

Table of Contents

Motivations

Motivations – What is the Network Coding?

"Coding at a node in a network…"

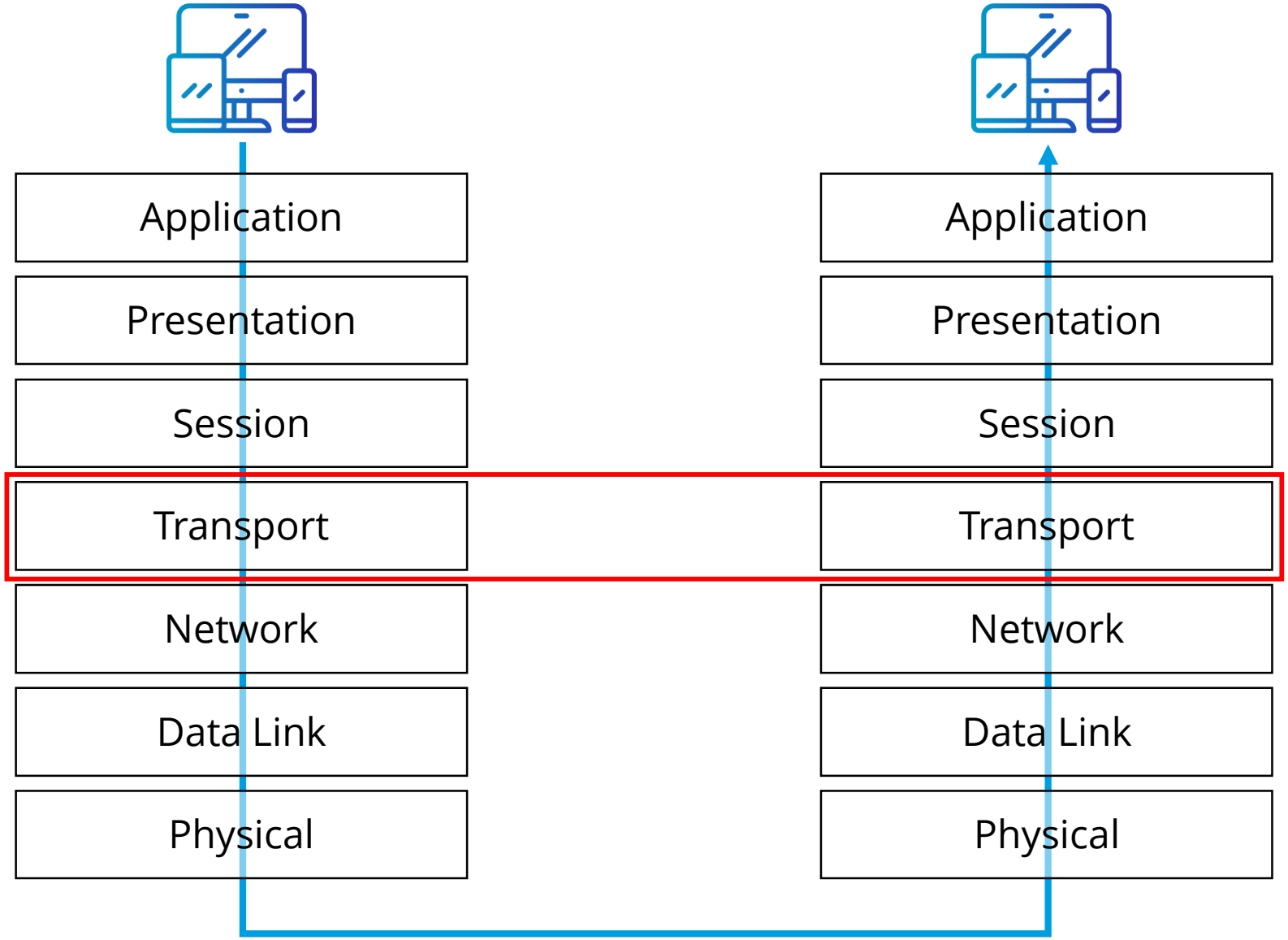
R. Ahlswede, *et al.*, "Network information flow," *IEEE Transactions on Information Theory , vol. 46, no. 4, Jul. 2000.*

Motivations – What is Coding?

*"Series of processes **to convert information into another form** or representation…"*

Motivations

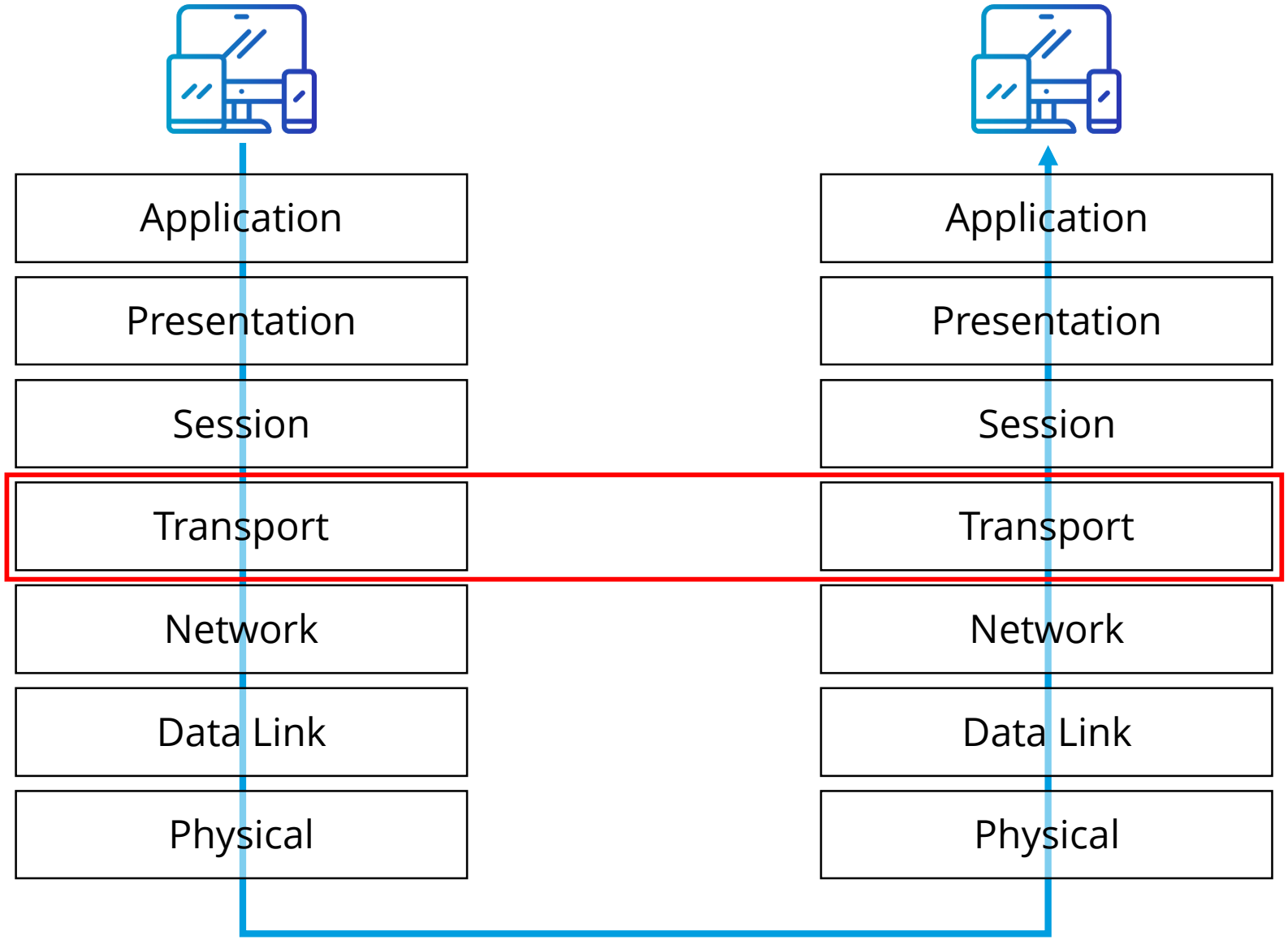
Network Coding such as RLNC



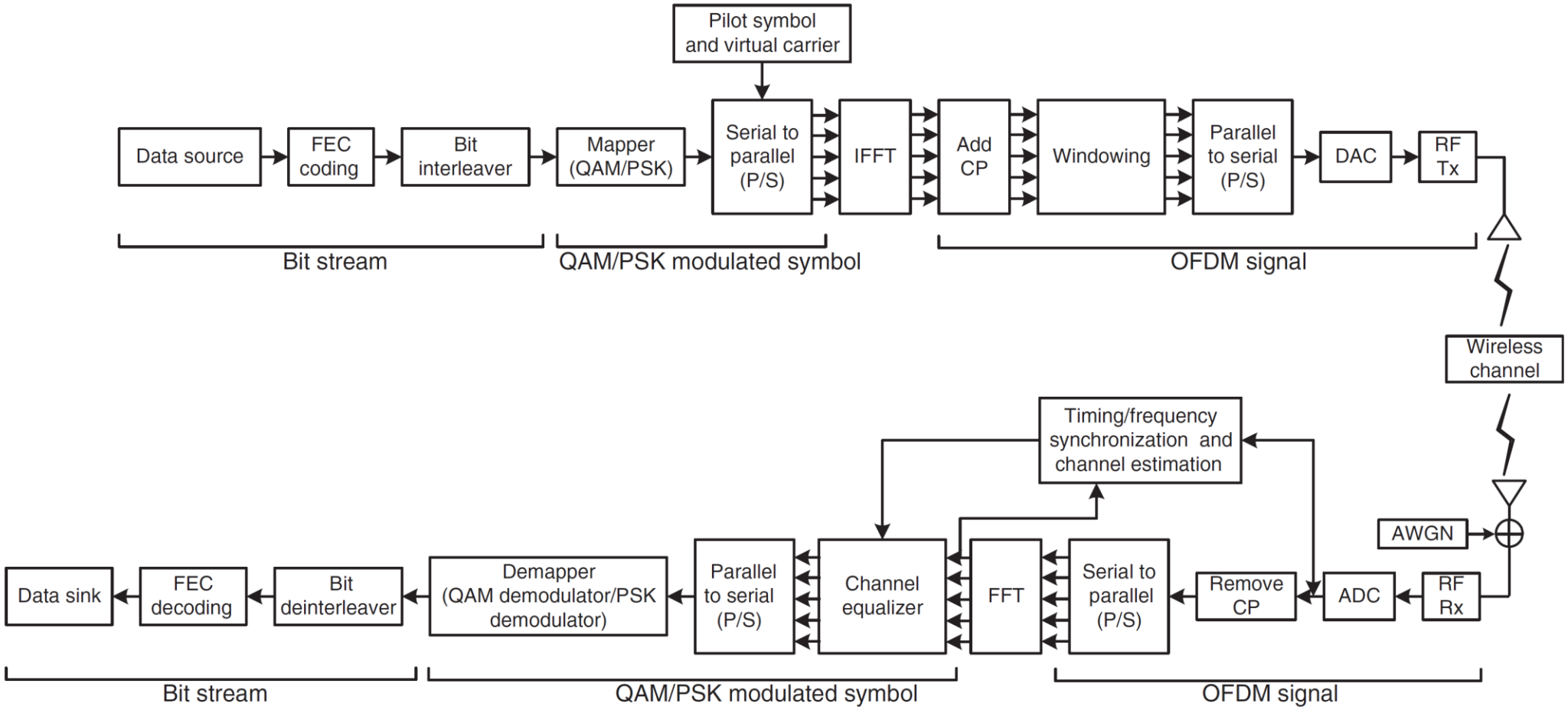
Motivations

Network Coding such as RLNC

Physical-layer NC (PNC) and Analog NC (ANC)

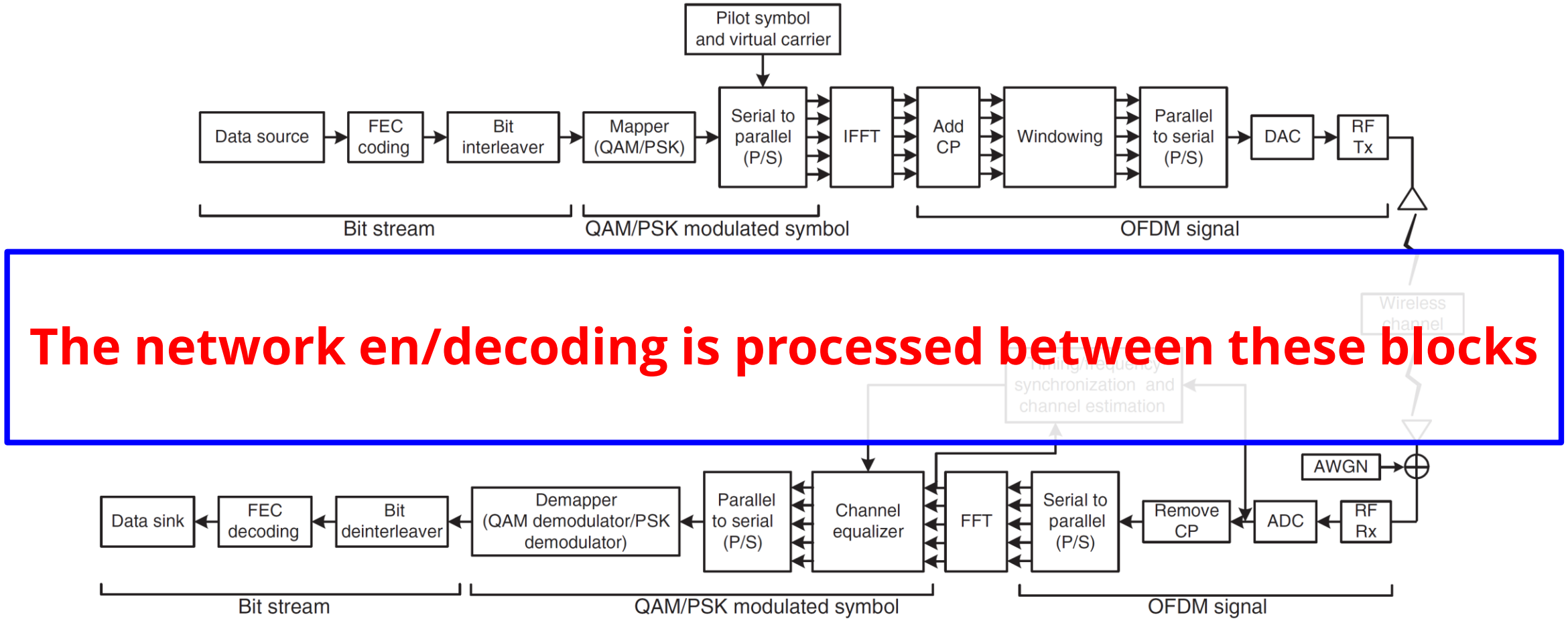


Motivations – Basic Physical Layer Blocks using OFDM



Y. S. Cho, *et al.*, "*MIMO-OFDM Wireless Communications with MATLAB*," Singapore: John Wiley & Sons (Asia) Pte Ltd, 210

Motivations – Basic Physical Layer Blocks using OFDM



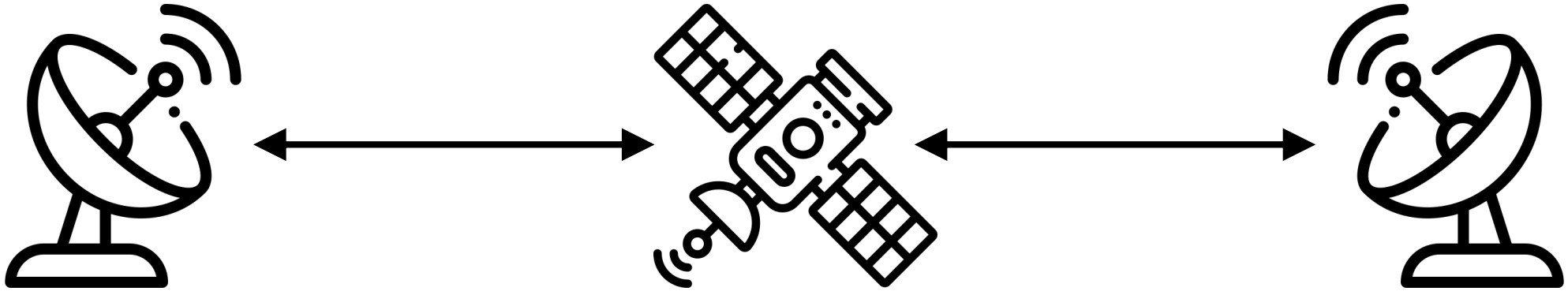
The network en/decoding is processed between these blocks

Y. S. Cho, *et al.*, "*MIMO-OFDM Wireless Communications with MATLAB*," Singapore: John Wiley & Sons (Asia) Pte Ltd, 210

Physical-layer Network Coding (PNC)

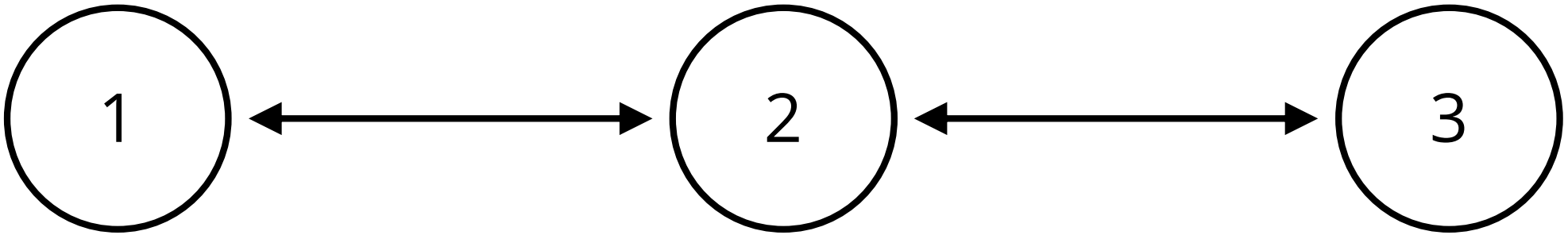
S. Zhang, S. C. Liew, and P. P. Lam, “Hot topic: Physical-layer network coding,” in *Proc. International Conference on Mobile Computing and Networking*, Sep. 2006.

Physical-layer Network Coding (PNC)



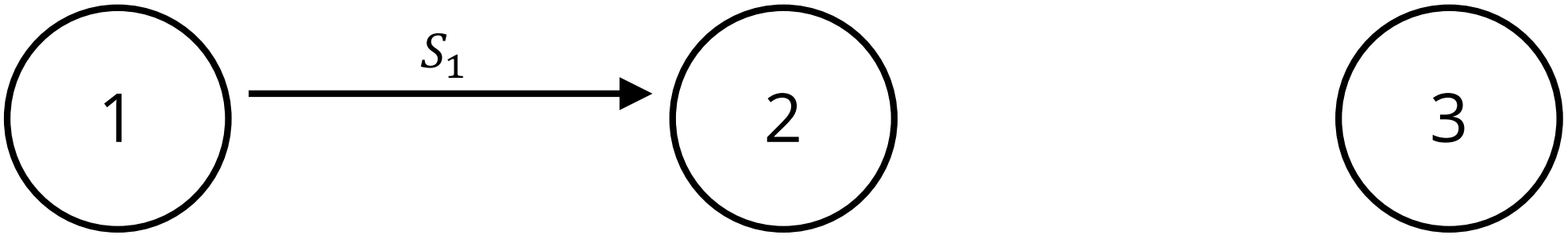
Two-way Relay Channel (TWRC)

Physical-layer Network Coding (PNC)

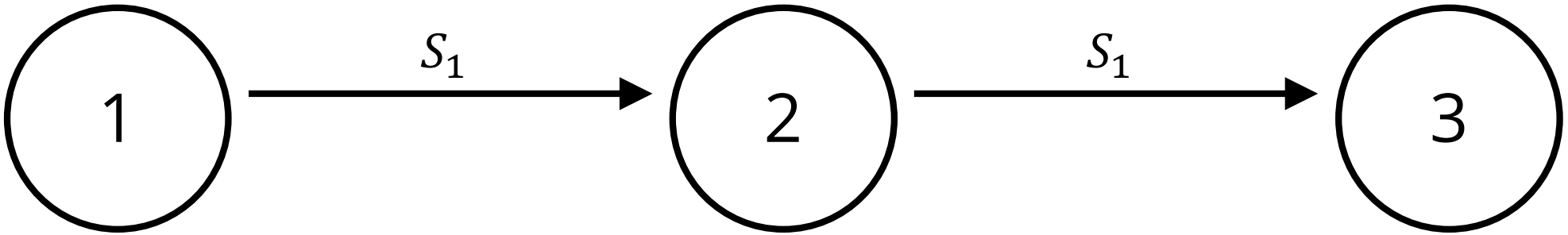


Two-way Relay Channel (TWRC)

Physical-layer Network Coding (PNC) – Straightforward Scheduling



Physical-layer Network Coding (PNC) – Straightforward Scheduling



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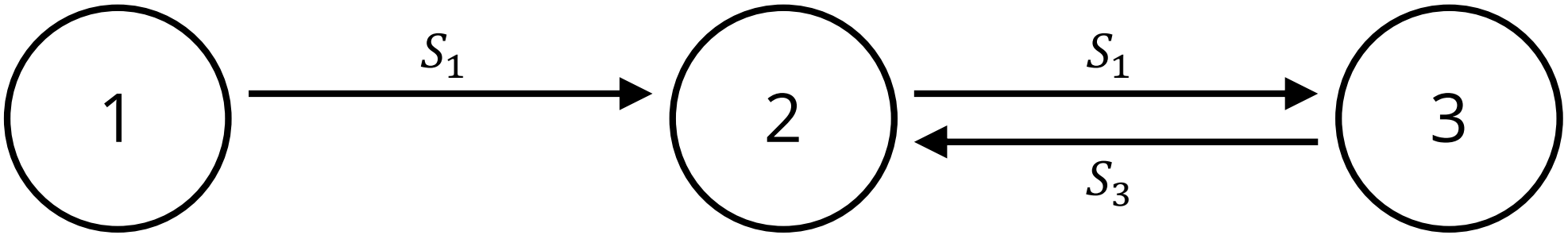

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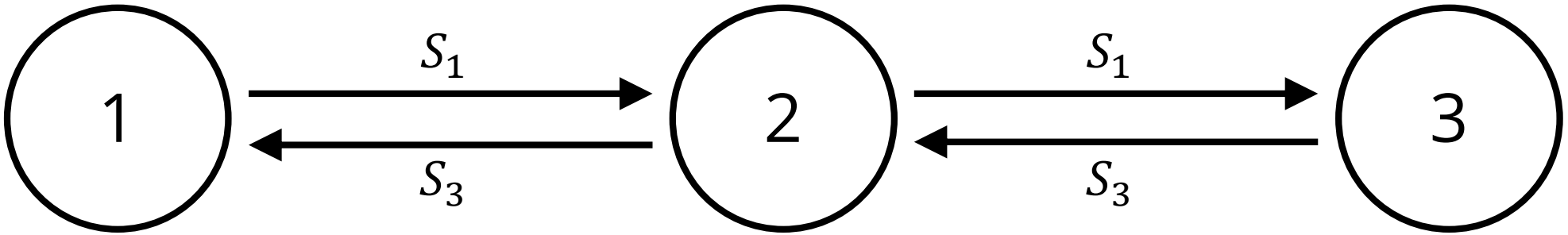
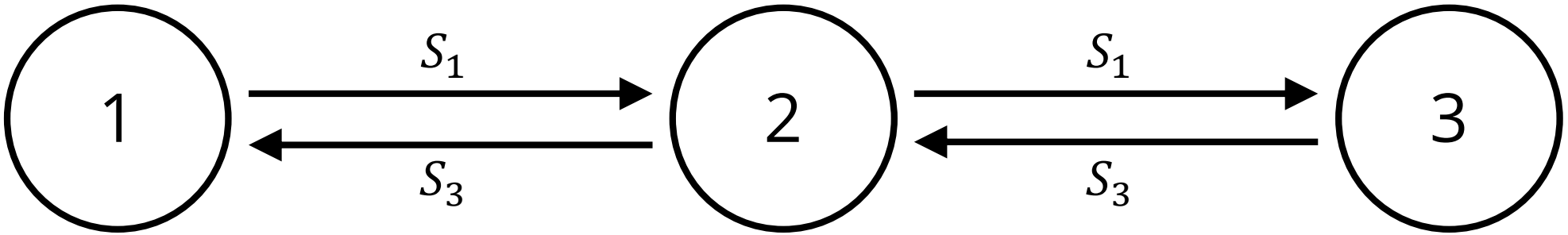
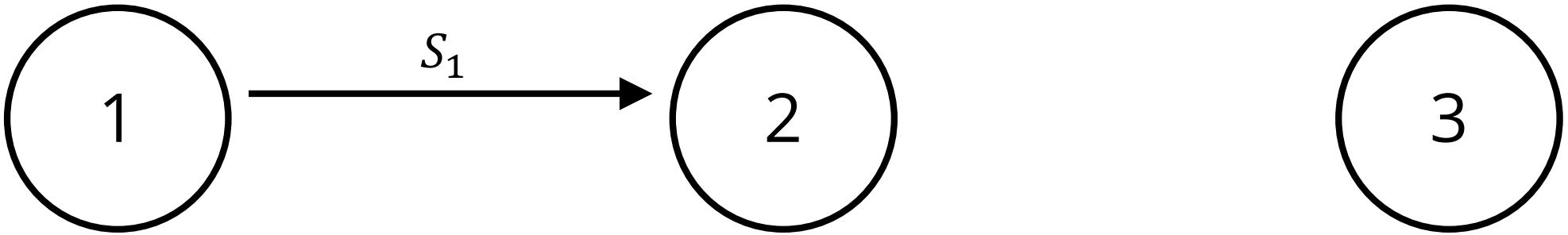


Physical-layer Network Coding (PNC) – Straightforward Scheduling

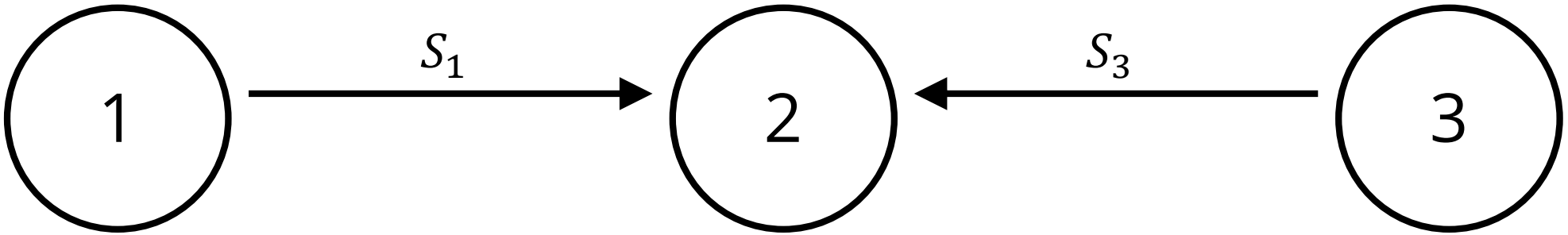


4 time slots are required to complete the transmission

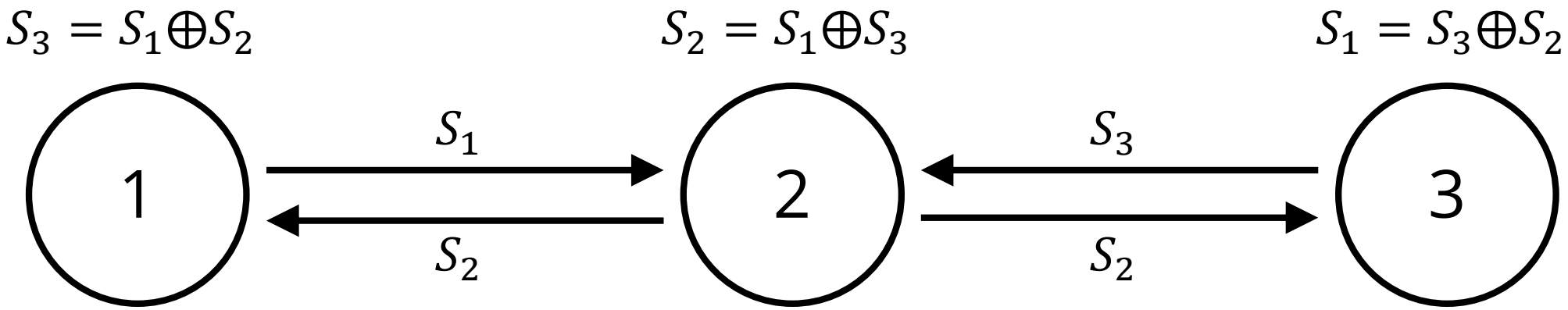
Physical-layer Network Coding (PNC) – Straightforward Network Coding



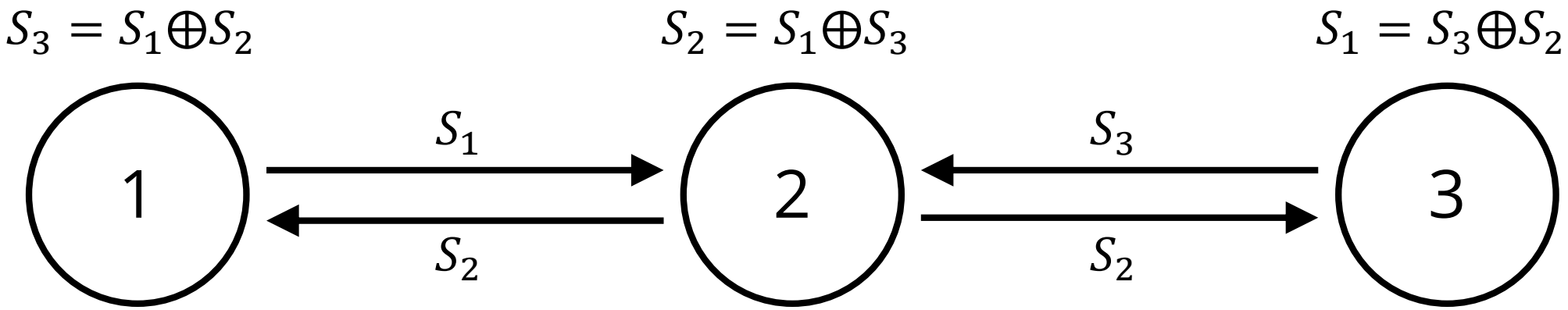
Physical-layer Network Coding (PNC) – Straightforward Network Coding



Physical-layer Network Coding (PNC) – Straightforward Network Coding



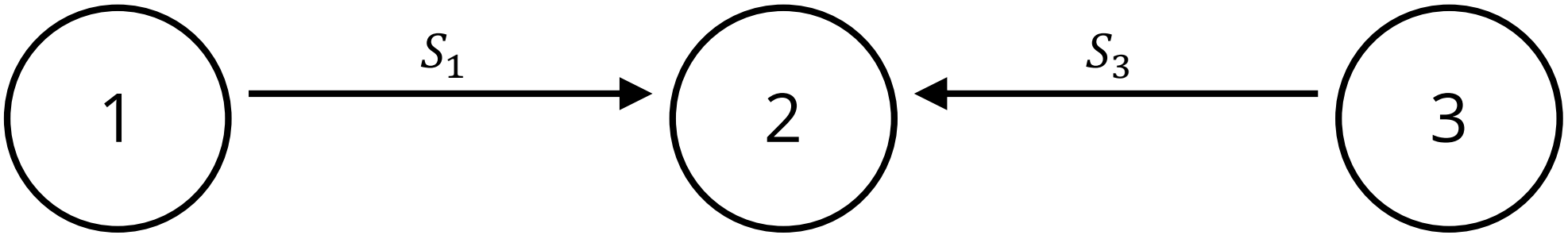
Physical-layer Network Coding (PNC) – Straightforward Network Coding



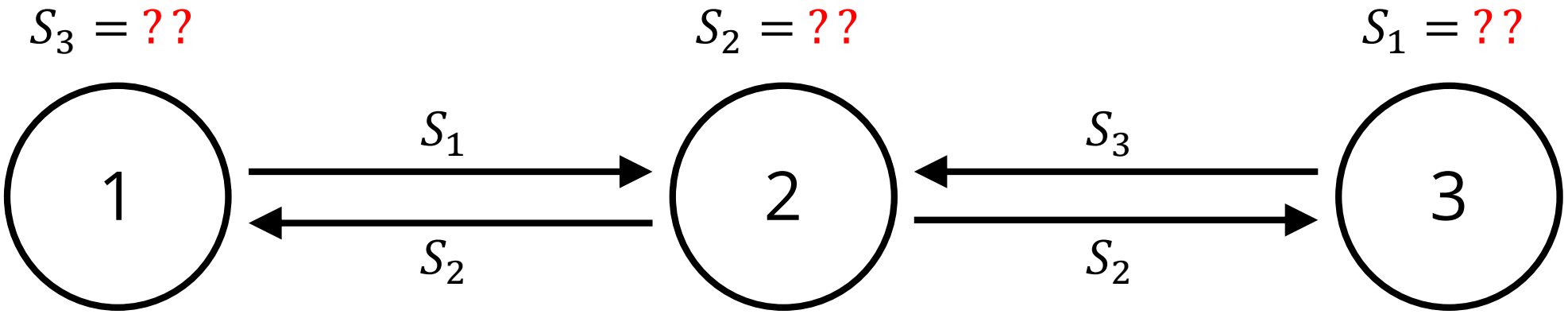
3 time slots are required to complete the transmission

(33% throughput improvement than the straightforward case)

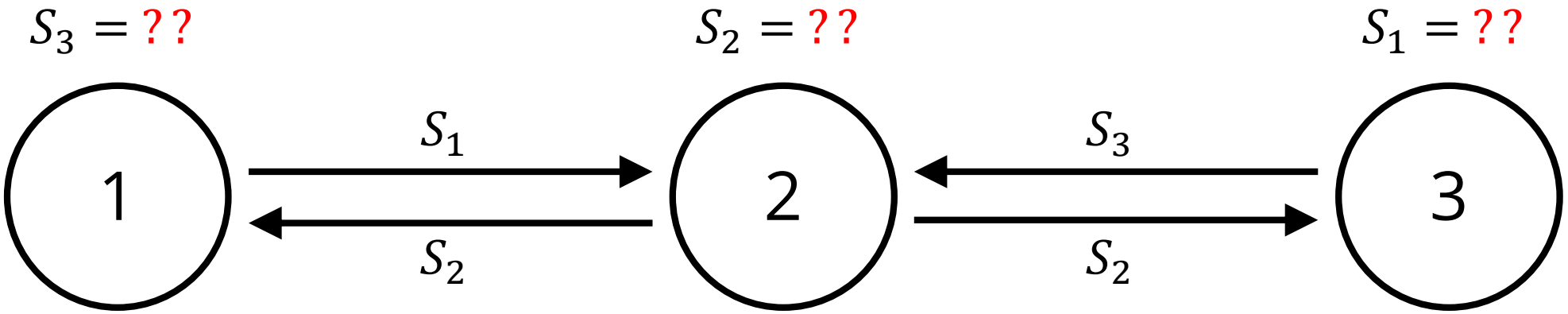
Physical-layer Network Coding (PNC)



Physical-layer Network Coding (PNC)



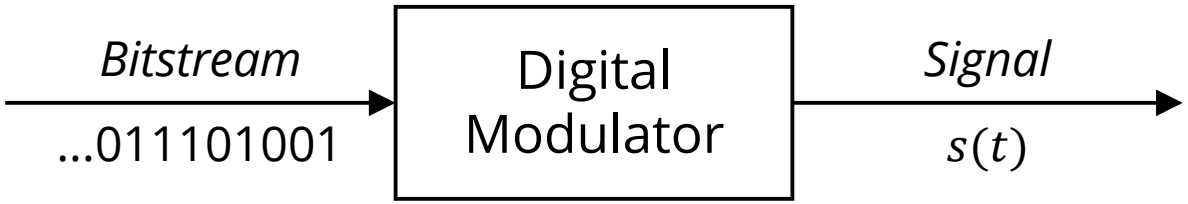
Physical-layer Network Coding (PNC)



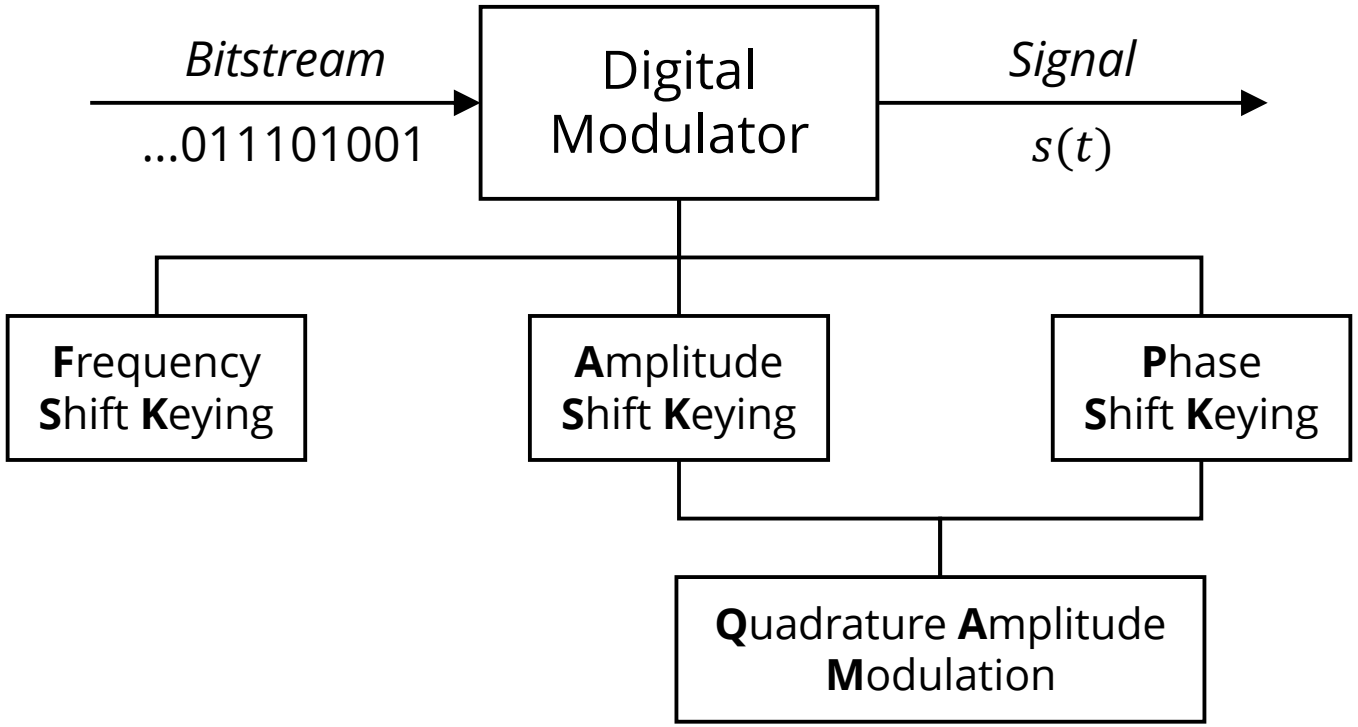
Only 2 time slots are required to complete the transmission

(100% throughput improvement than the straightforward case)

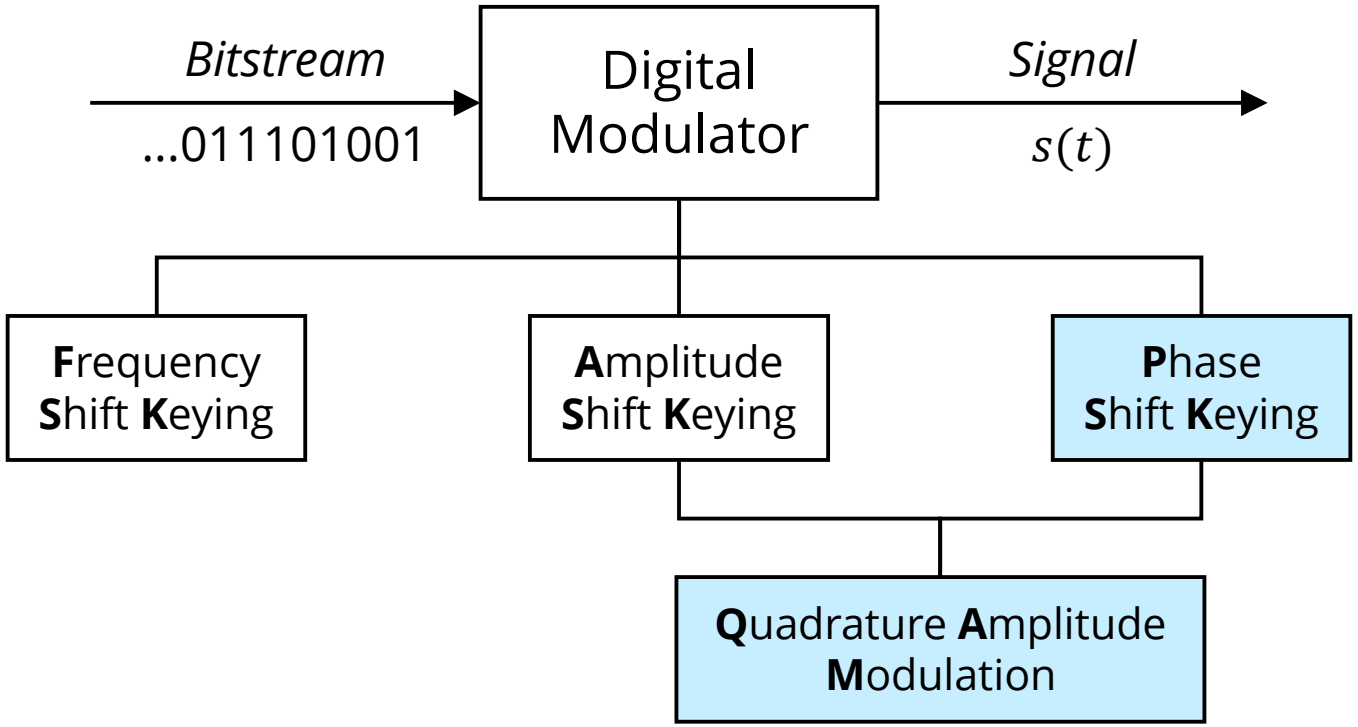
Physical-layer Network Coding (PNC) – Digital Modulation



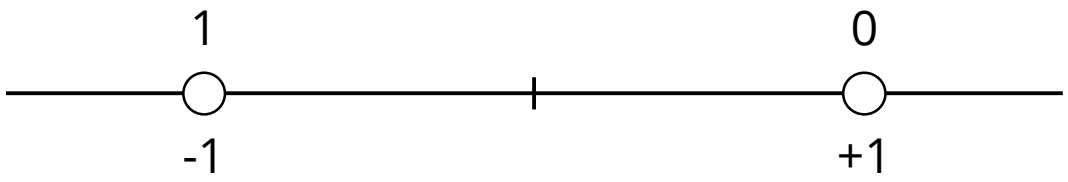
Physical-layer Network Coding (PNC) – Digital Modulation



Physical-layer Network Coding (PNC) – Digital Modulation

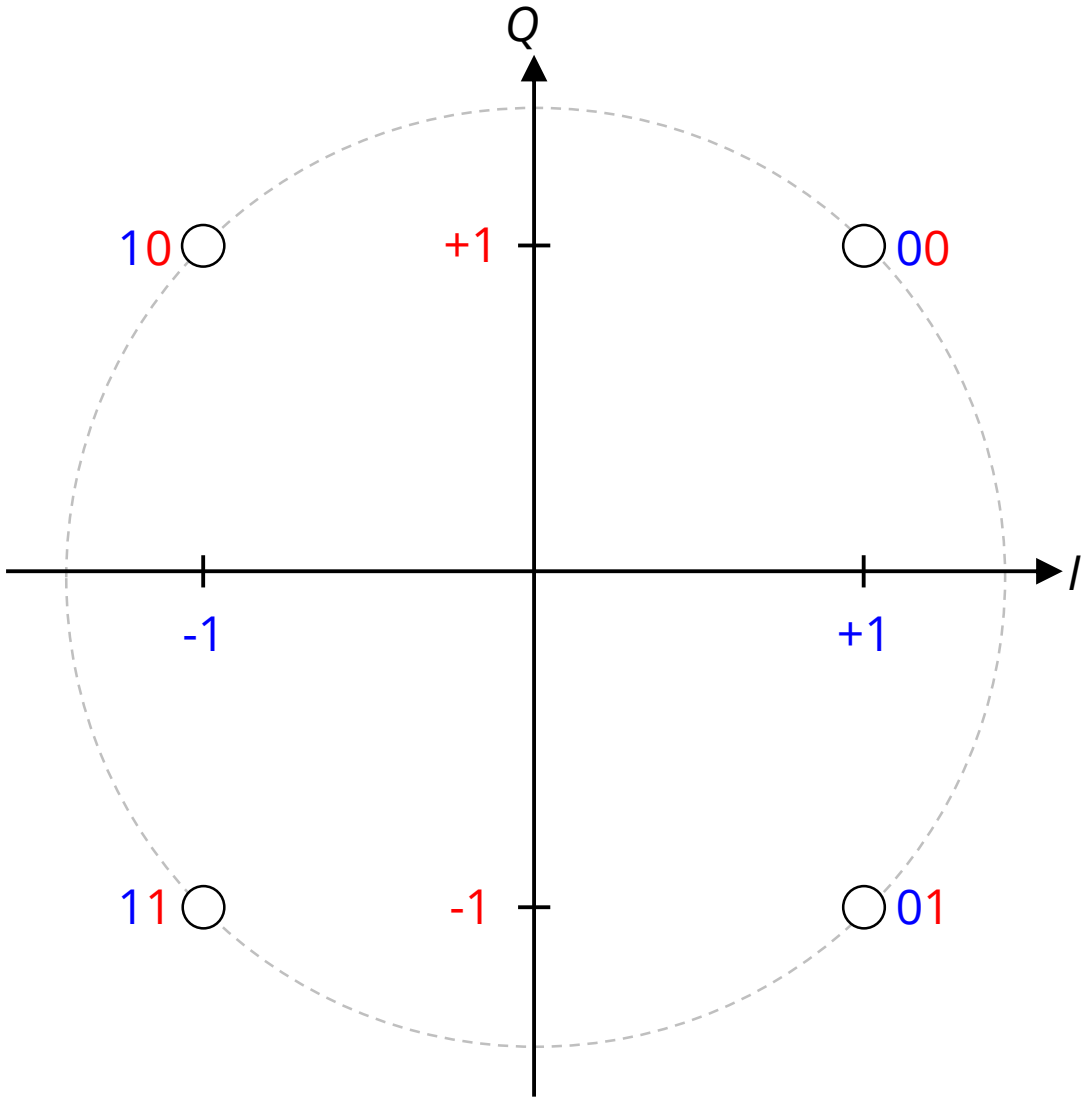


Physical-layer Network Coding (PNC) – Binary PSK (BPSK)



BPSK	
Bit	**Symbol**
0	+1
1	-1

Physical-layer Network Coding (PNC) – Quadrature PSK (QPSK)

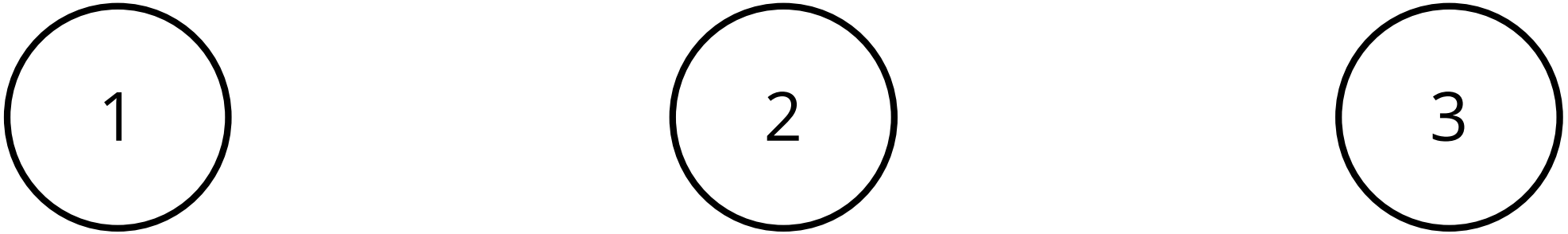


QPSK			
Bit		Symbol	
I	*Q*	*I*	*Q*
0	0	+1	+1
0	1	+1	-1
1	0	-1	+1
1	1	-1	-1

I and Q denote In-phase and Quadrature, respectively

$$s(t) = a\cos(\omega t) - b\sin(\omega t)$$

Physical-layer Network Coding (PNC) – Straightforward Network Coding



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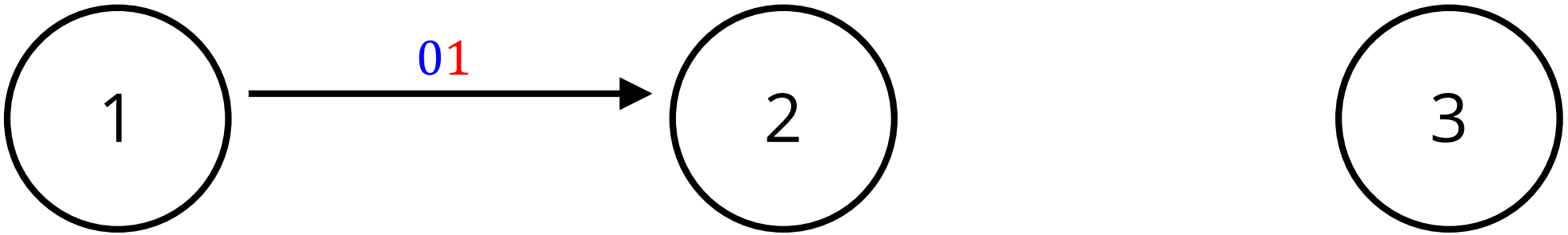

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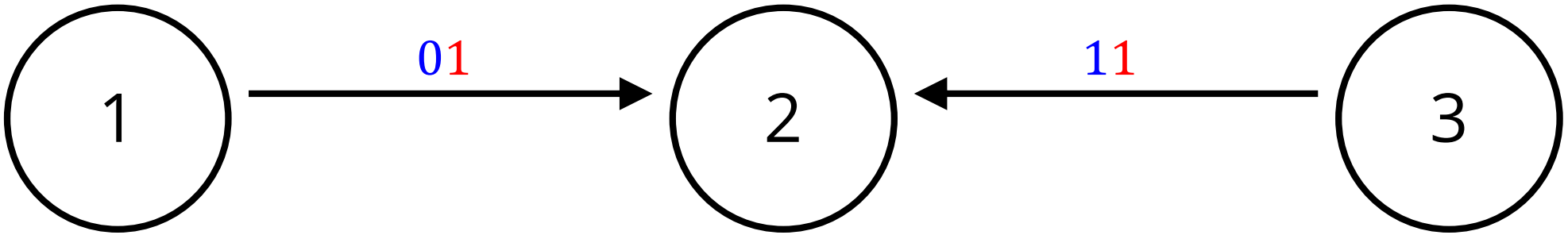

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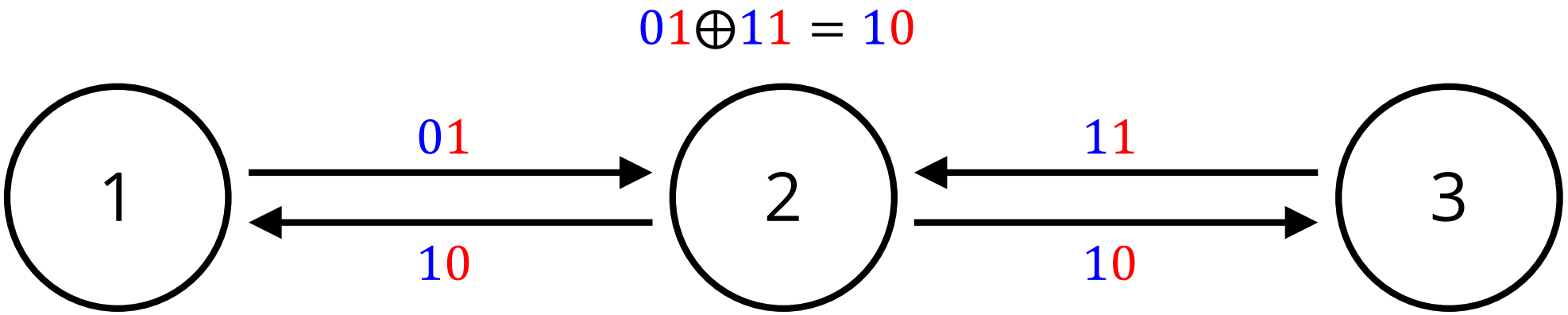

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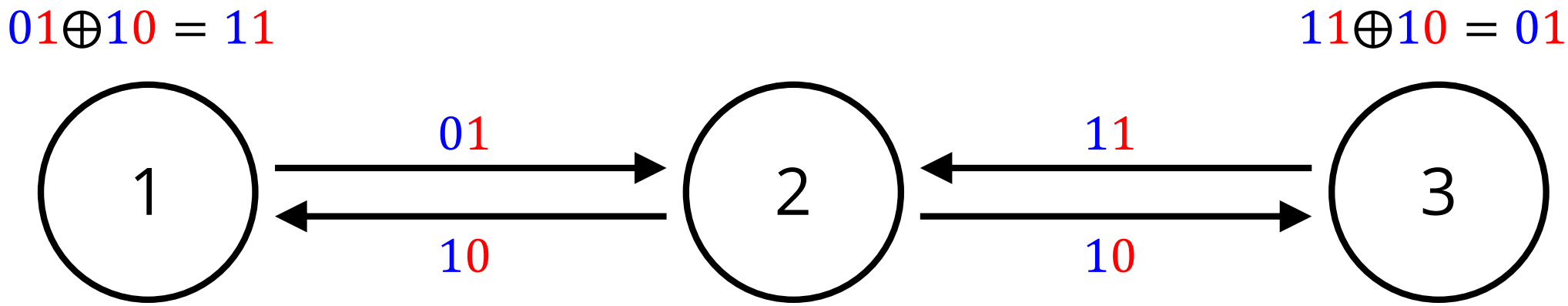
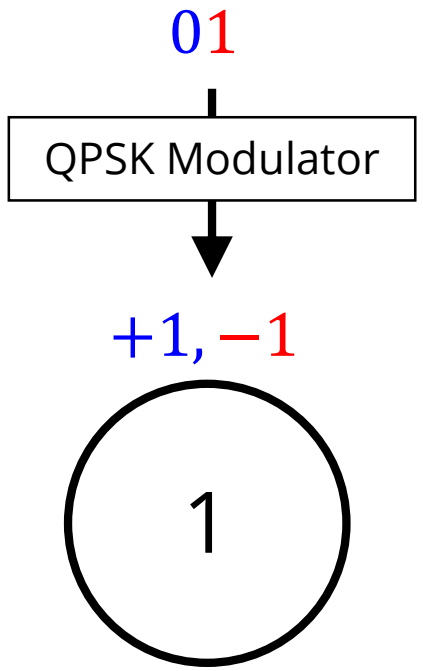
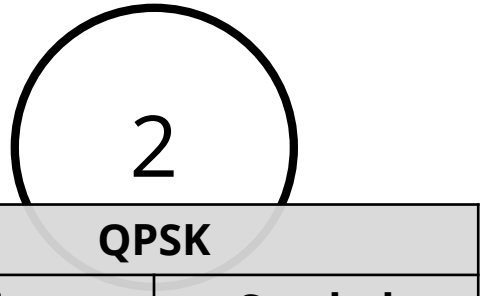
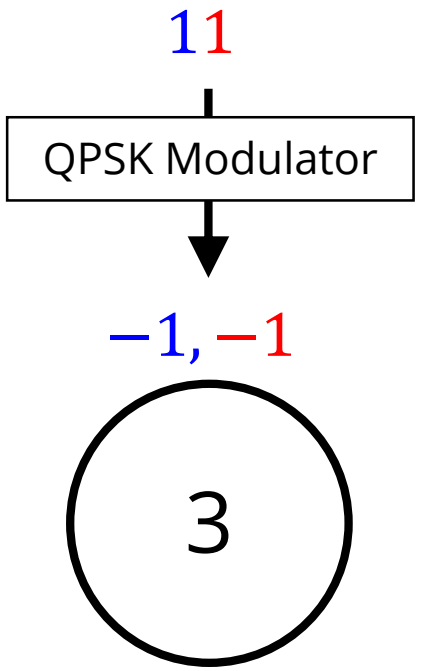


Physical-layer Network Coding (PNC) – Encoding

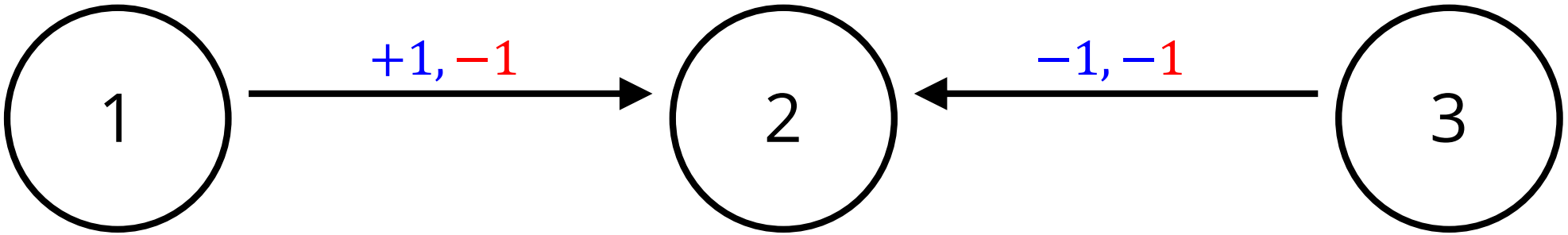




QPSK			
Bit		Symbol	
I	*Q*	*I*	*Q*
0	0	+1	+1
0	1	+1	-1
1	0	-1	+1
1	1	-1	-1



Physical-layer Network Coding (PNC) – Encoding



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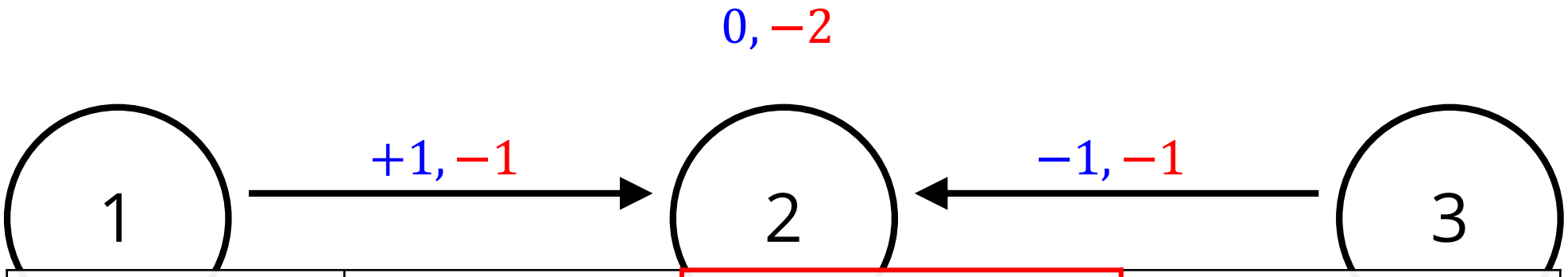
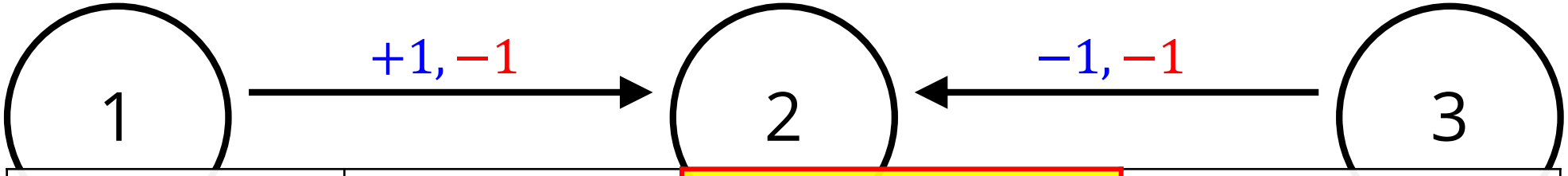


Symbol from Node 1 a_1 or b_1	Symbol from Node 3 a_3 or b_3	Superposed symbol received at Node 2 $a_1 + a_3$ or $b_1 + b_3$	Mapping to symbol to be transmitted by Node 2 a_2 or b_2
+1	+1	+2	+1
+1	-1	0	-1
-1	+1	0	-1
-1	-1	-2	+1

Physical-layer Network Coding (PNC) – Encoding

$$y(t) = s_1(t) + s_2(t)$$
$$= [a_1 \cos(\omega t) - b_1 \sin(\omega t)] + [a_2 \cos(\omega t) - b_2 \sin(\omega t)]$$
$$= (a_1 + a_2)\cos(\omega t) - (b_1 + b_2)\sin(\omega t)$$

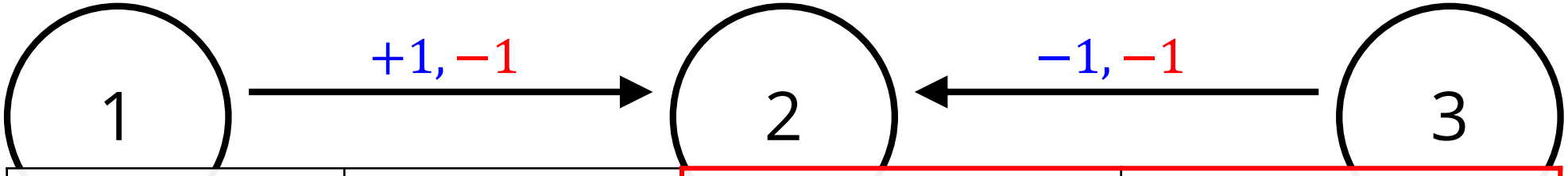


Symbol from Node 1 a_1 or b_1	Symbol from Node 3 a_3 or b_3	Superposed symbol received at Node 2 $a_1 + a_3$ or $b_1 + b_3$	Mapping to symbol to be transmitted by Node 2 a_2 or b_2
+1	+1	+2	+1
+1	-1	0	-1
-1	+1	0	-1
-1	-1	-2	+1

Physical-layer Network Coding (PNC) – Encoding

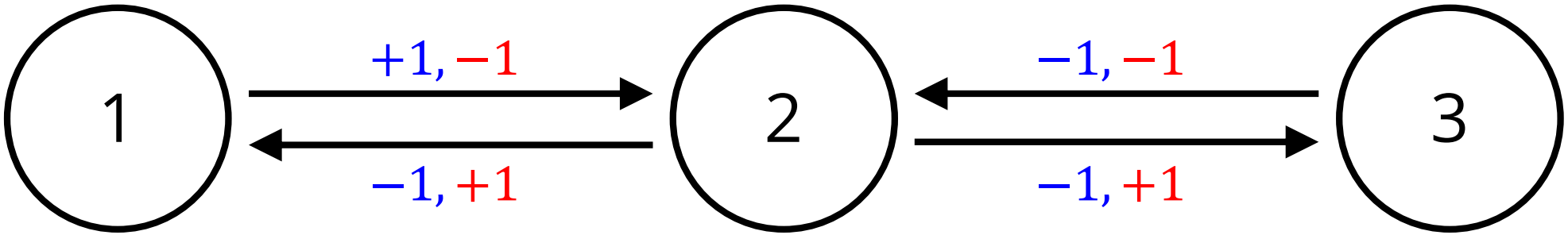
$$+1 + (-1) = 0 \rightarrow -1$$

$$-1 + (-1) = -2 \rightarrow +1$$



Symbol from Node 1 a_1 or b_1	Symbol from Node 3 a_3 or b_3	Superposed symbol received at Node 2 $a_1 + a_3$ or $b_1 + b_3$	Mapping to symbol to be transmitted by Node 2 a_2 or b_2
+1	+1	+2	+1
+1	-1	0	-1
-1	+1	0	-1
-1	-1	-2	+1

Physical-layer Network Coding (PNC) – Encoding



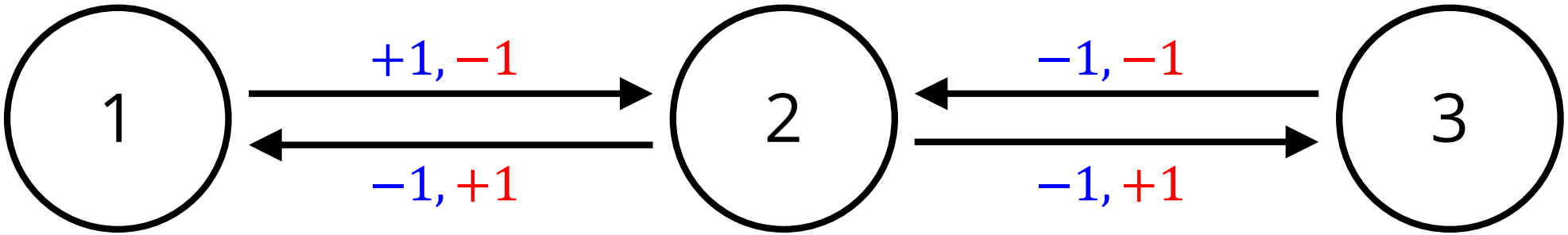
Physical-layer Network Coding (PNC) – Decoding

$+1-1=0$

$-1+1=0$

$-1-1=-2$

$-1+1=0$



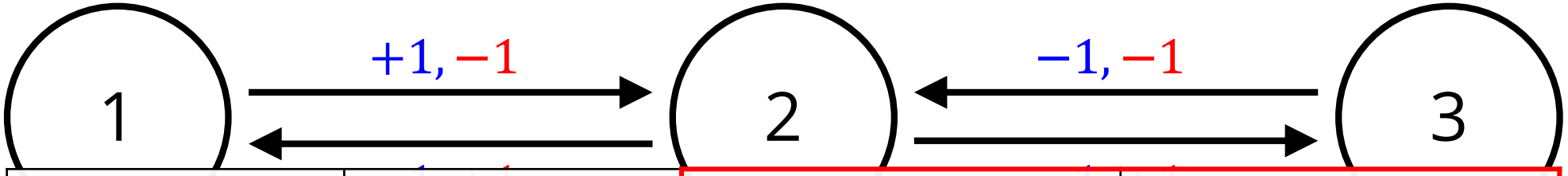
Physical-layer Network Coding (PNC) – Decoding

$+1-1=0 \rightarrow -1$

$-1+1=0 \rightarrow -1$

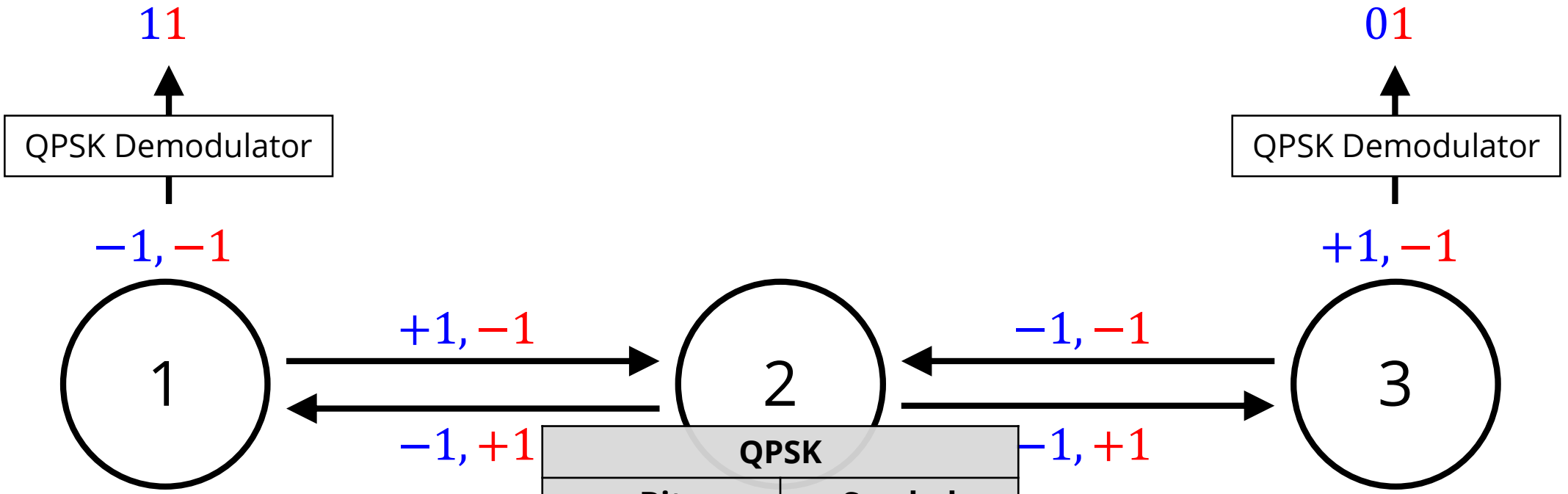
$-1-1=-2 \rightarrow +1$

$-1+1=0 \rightarrow -1$



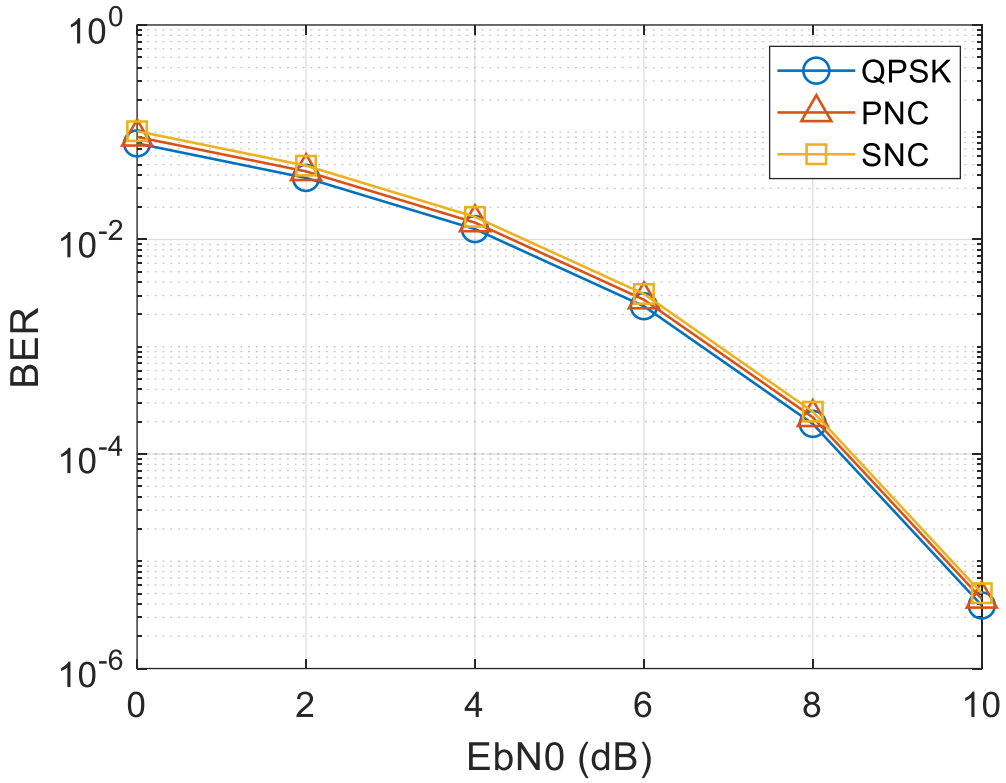
Symbol from Node 1 a_1 or b_1	Symbol from Node 3 a_3 or b_3	Composite symbol received at Node 2 $a_1 + a_3$ or $b_1 + b_3$	Mapping to symbol to be transmitted by Node 2 a_2 or b_2
+1	+1	+2	+1
+1	-1	0	-1
-1	+1	0	-1
-1	-1	-2	+1

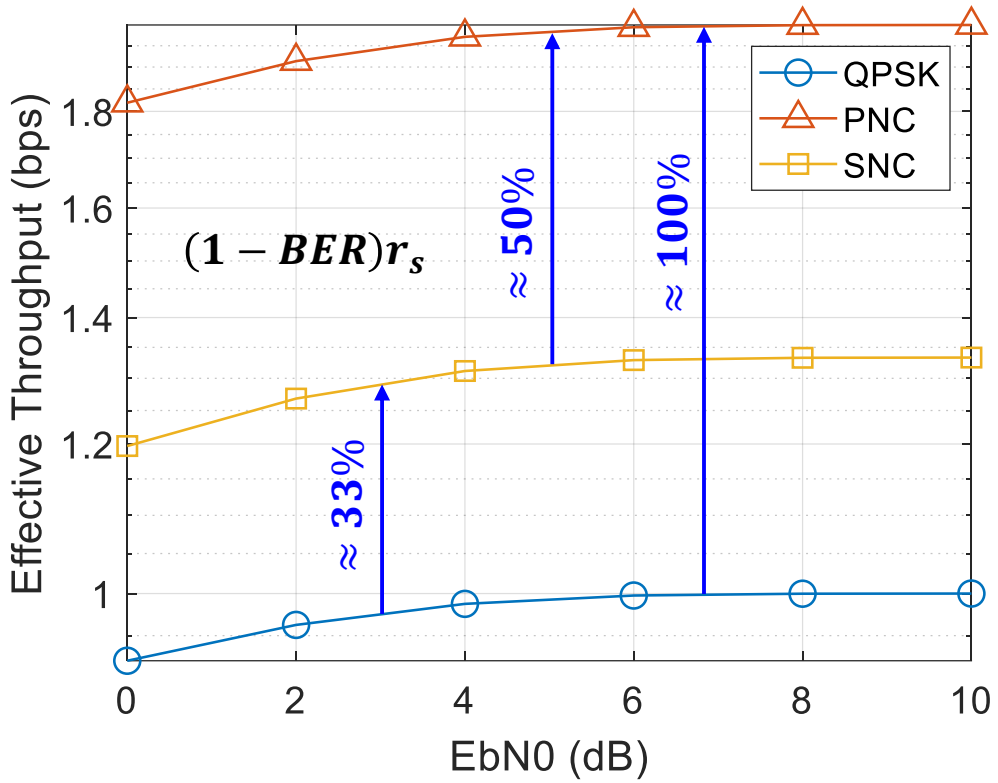
Physical-layer Network Coding (PNC) – Decoding



QPSK			
Bit		Symbol	
I	*Q*	*I*	*Q*
0	0	+1	+1
0	1	+1	-1
1	0	-1	+1
1	1	-1	-1

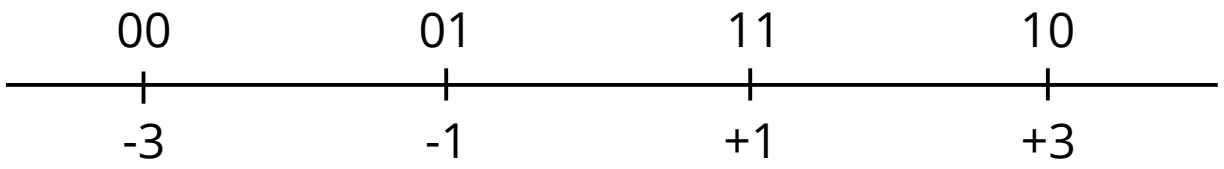
Physical-layer Network Coding (PNC) – Idle Performance using QPSK



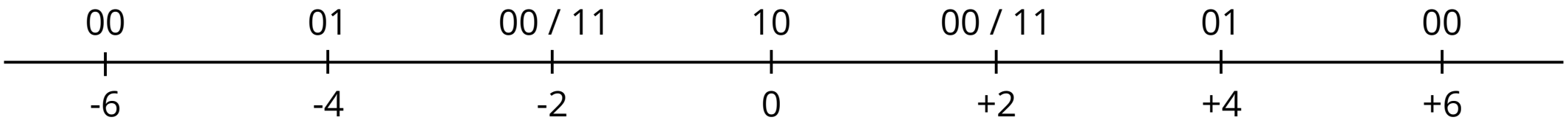


Physical-layer Network Coding (PNC) – Chronic Problem

I-axis of 16QAM (i.e., 4ASK)

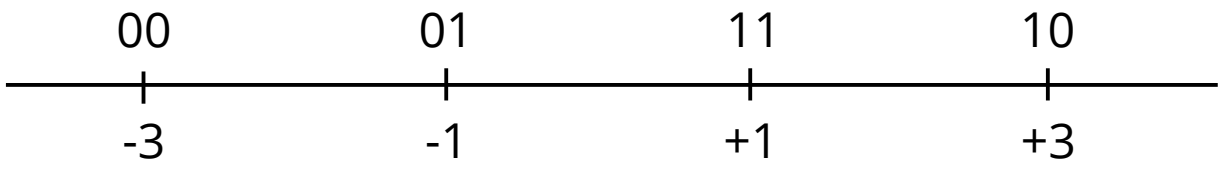


Superposed Constellation

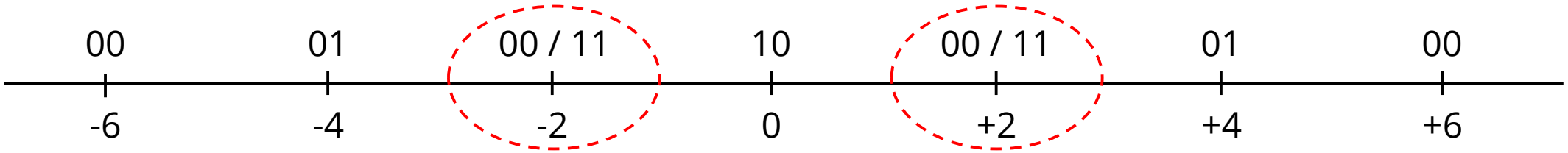


Physical-layer Network Coding (PNC) – Chronic Problem

I-axis of 16QAM (i.e., 4ASK)



Composite Constellation



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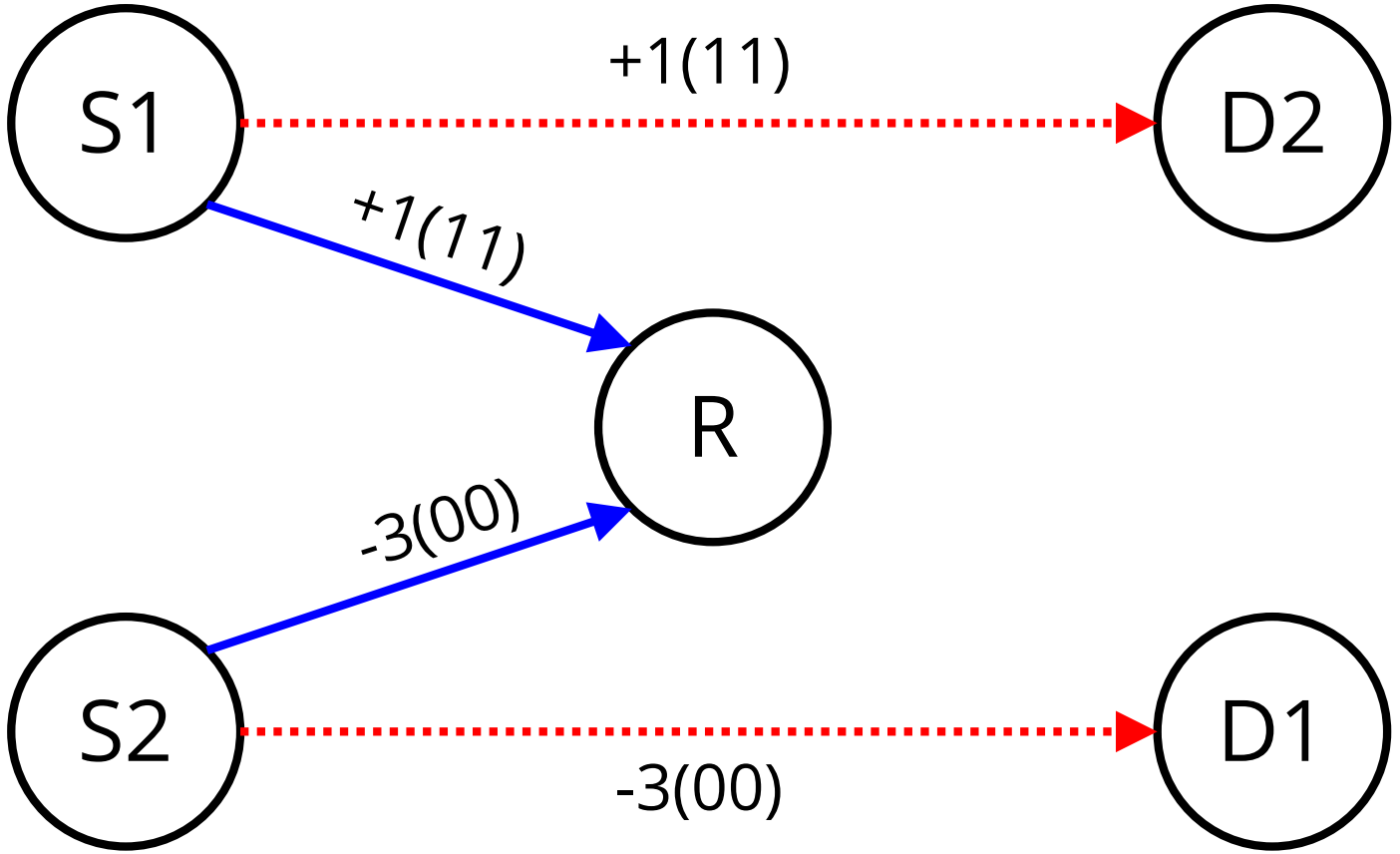
Physical-layer Network Coding (PNC) – One Relay Butterfly

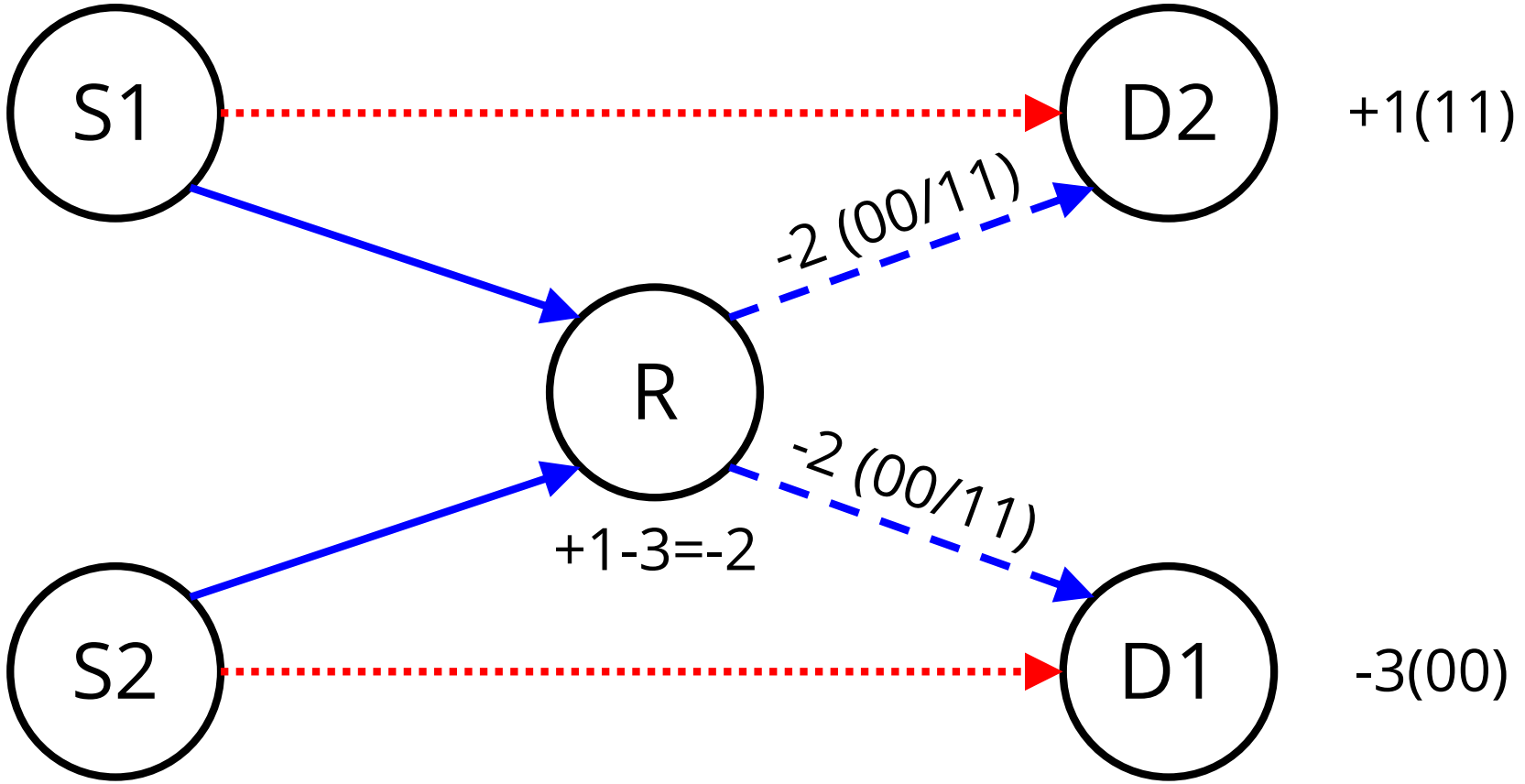


Intended Signal (1st Time Slot)

Interference Signal (1nd Time Slot)

Intended Signal (2nd Time Slot)

Physical-layer Network Coding (PNC) – One Relay Butterfly

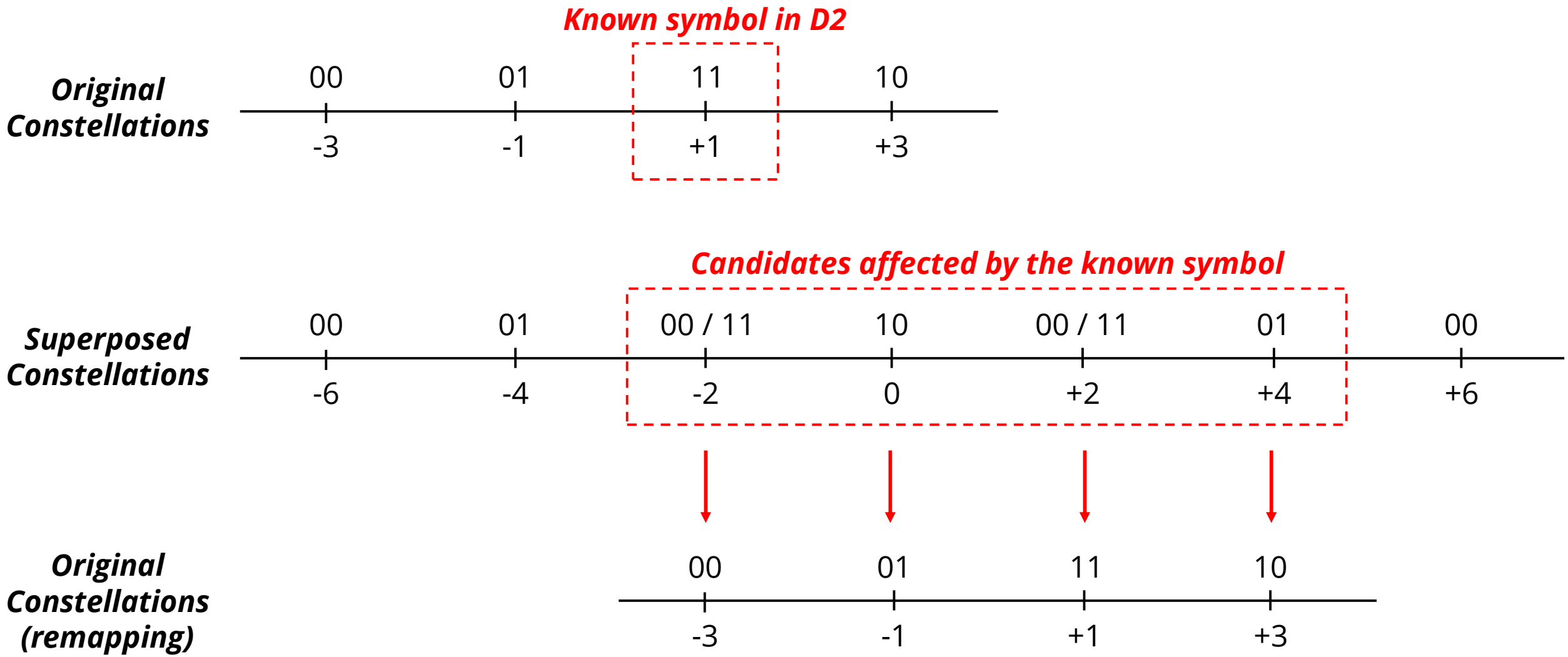


Intended Signal (1st Time Slot)

Interference Signal (1nd Time Slot)

Intended Signal (2nd Time Slot)

Physical-layer Network Coding (PNC) – One Relay Butterfly



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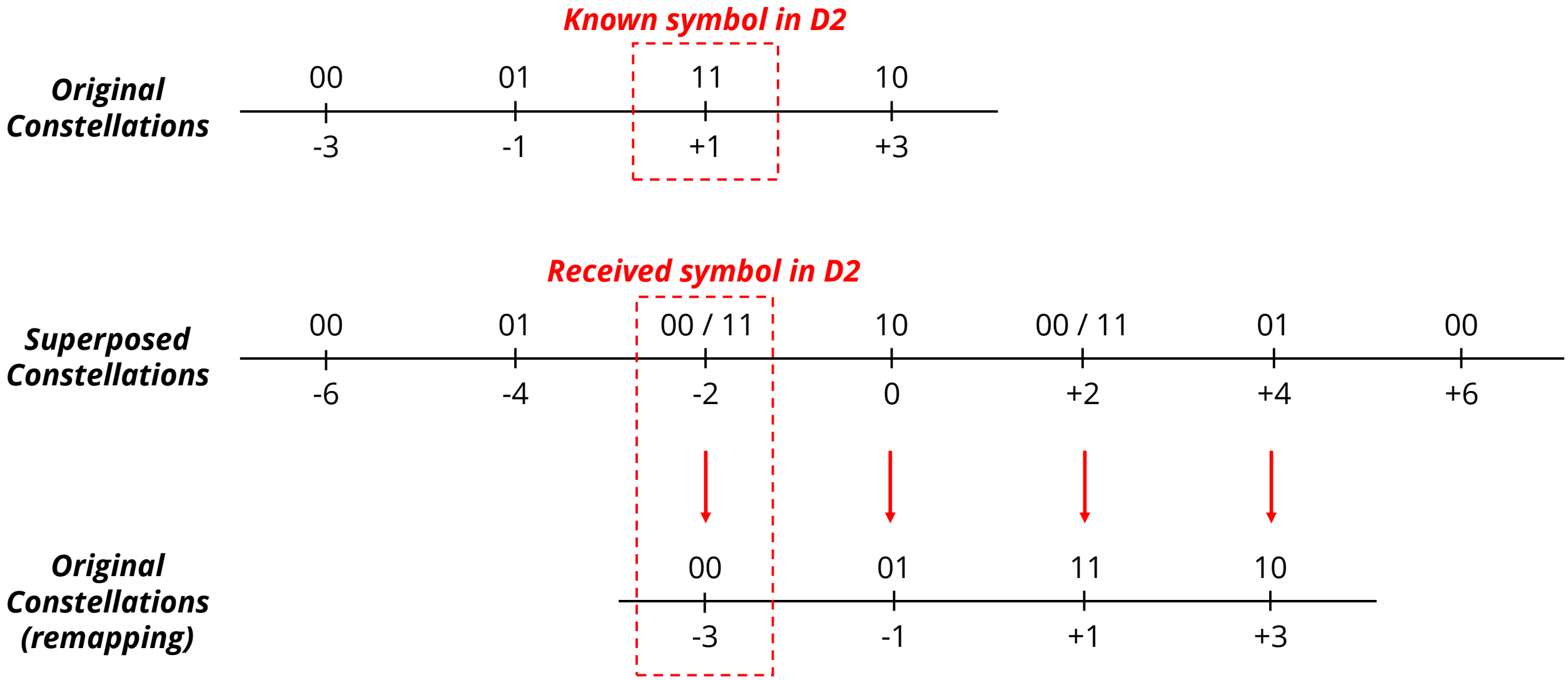
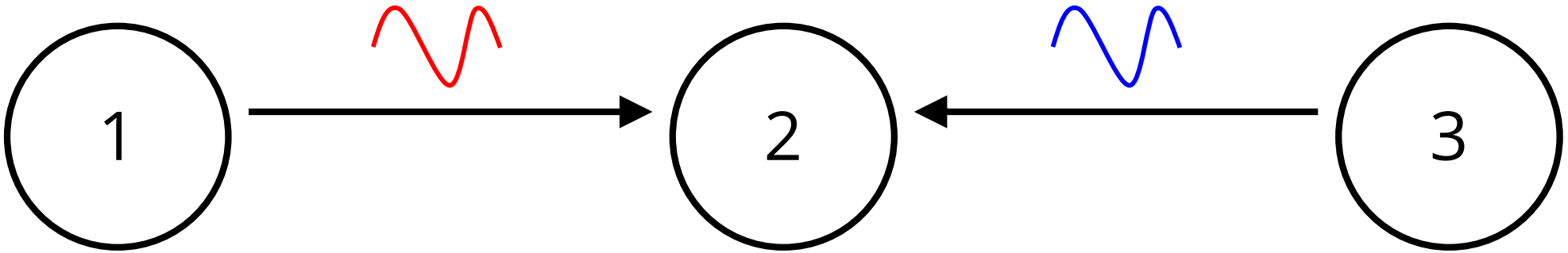


Analog Network Coding (ANC)

S. Katti, S. Gollakota, and D. Katabi, “Embracing wireless Interference: Analog network coding,” in *Proc. ACM SIGCOMM Computer Communication Review*, Aug. 2007.

Analog Network Coding (ANC) – Concept



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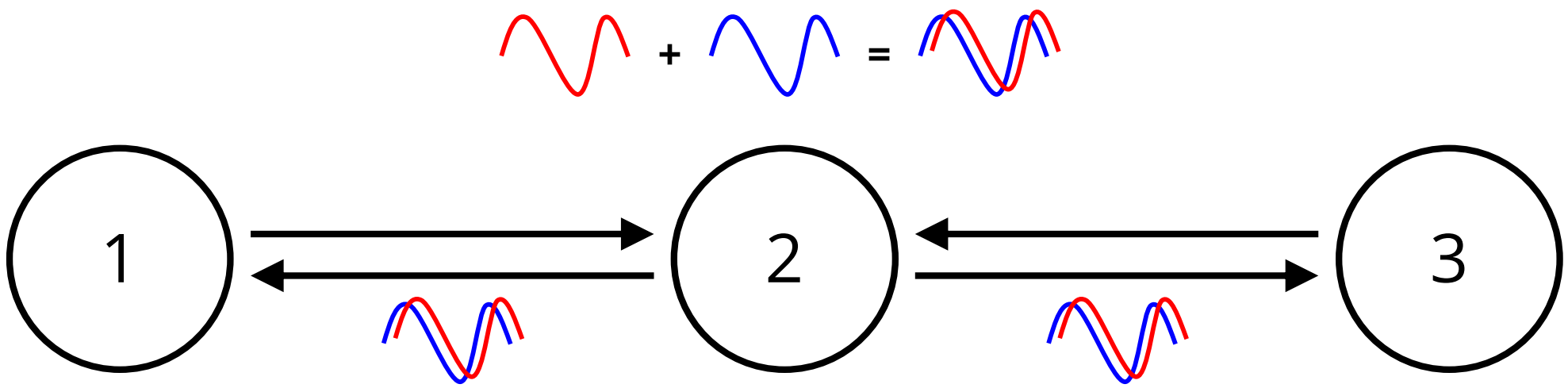
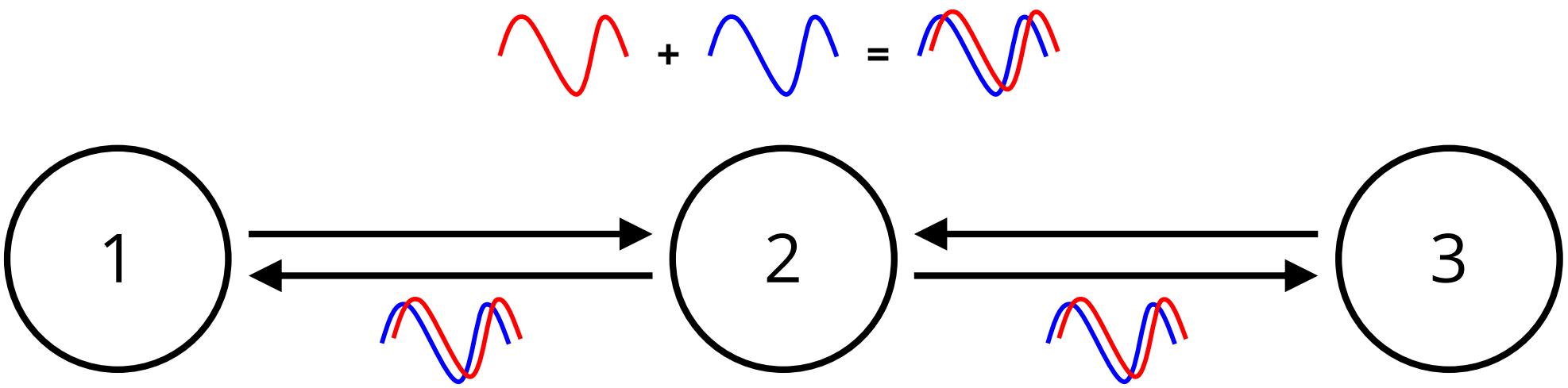
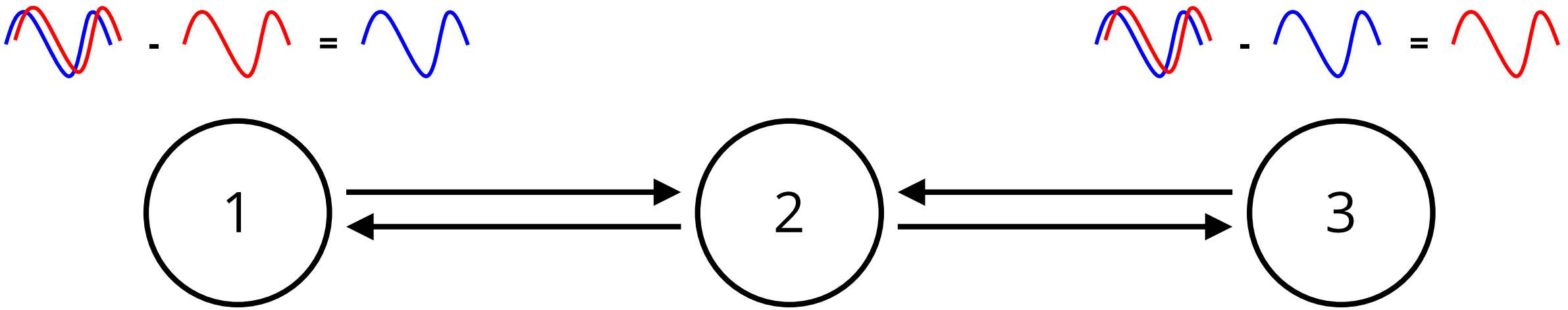


Analog Network Coding (ANC) – Concept

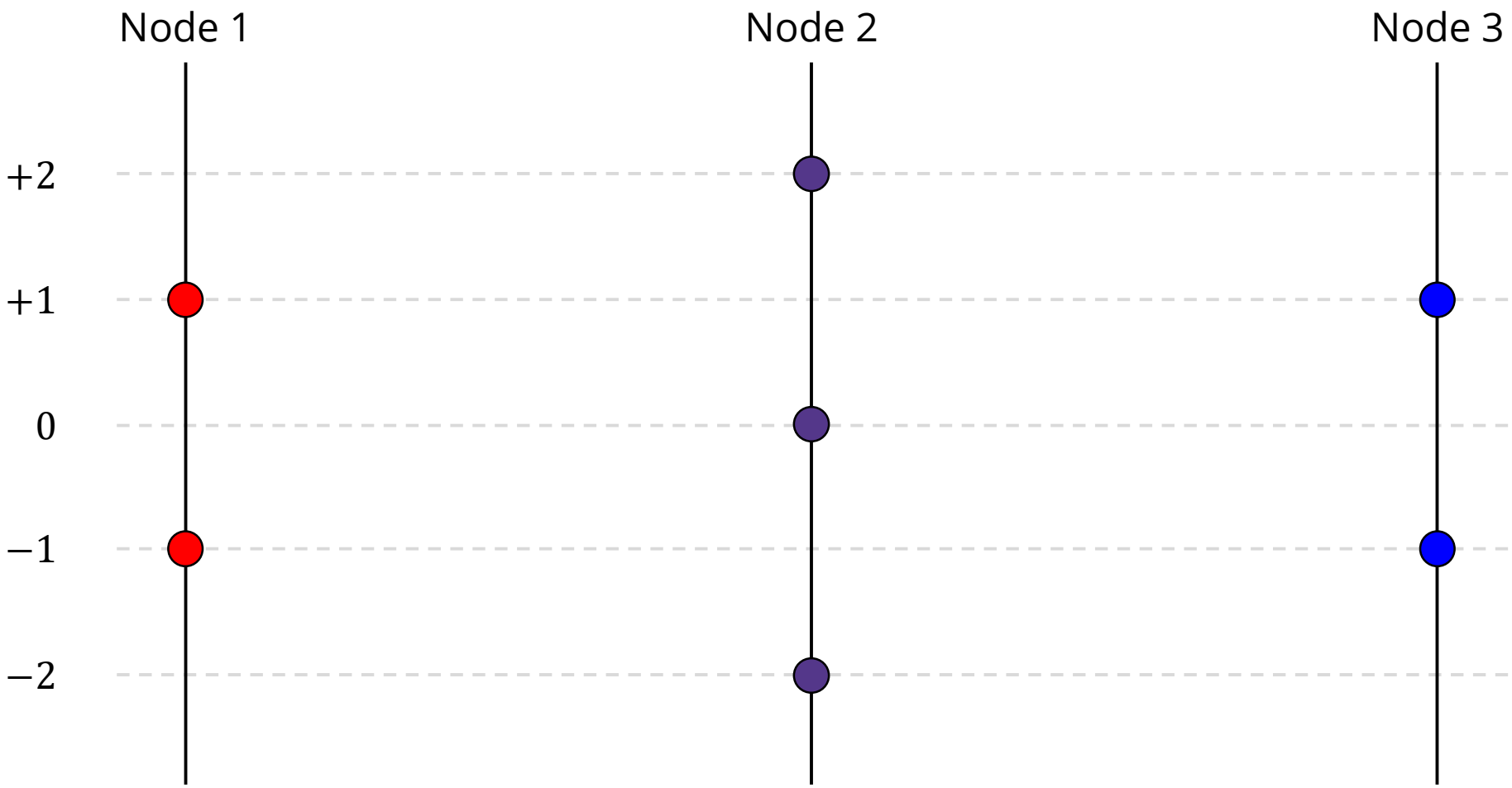


It does like the amplify-and-forward (AF) relay

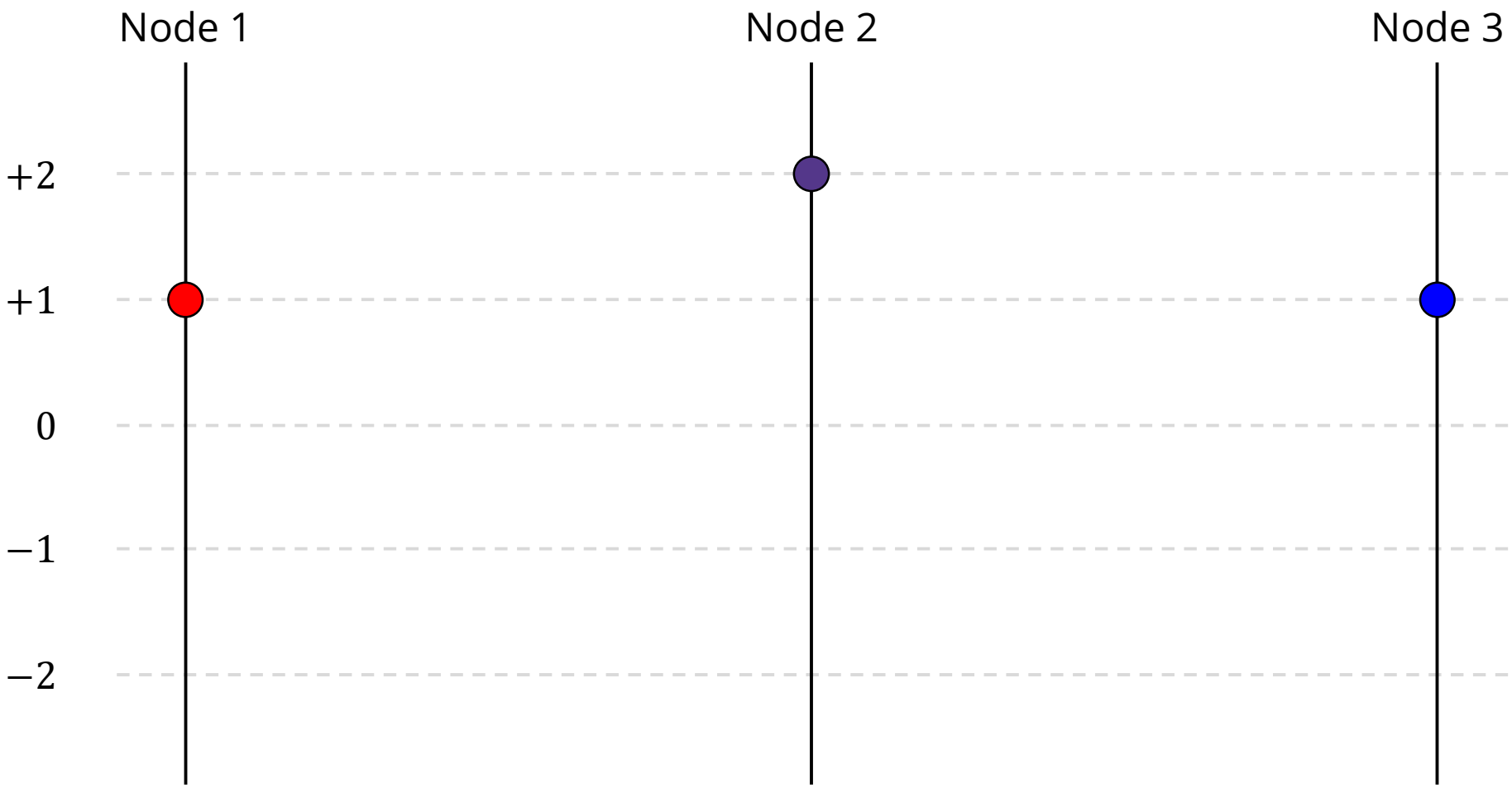
Analog Network Coding (ANC) – Concept



Analog Network Coding (ANC) – BPSK Example



Analog Network Coding (ANC) – BPSK Example (1st Time Slot)



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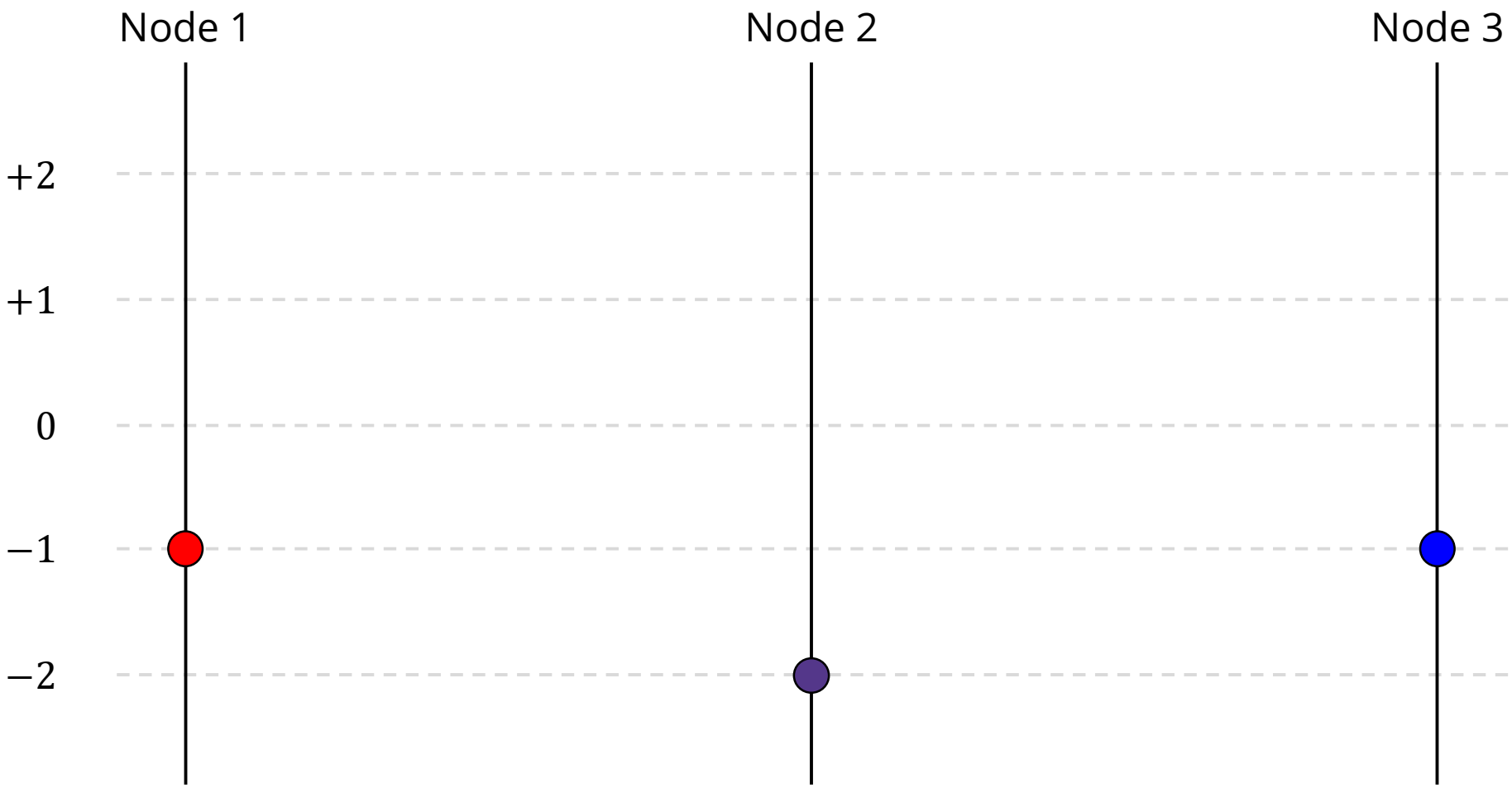

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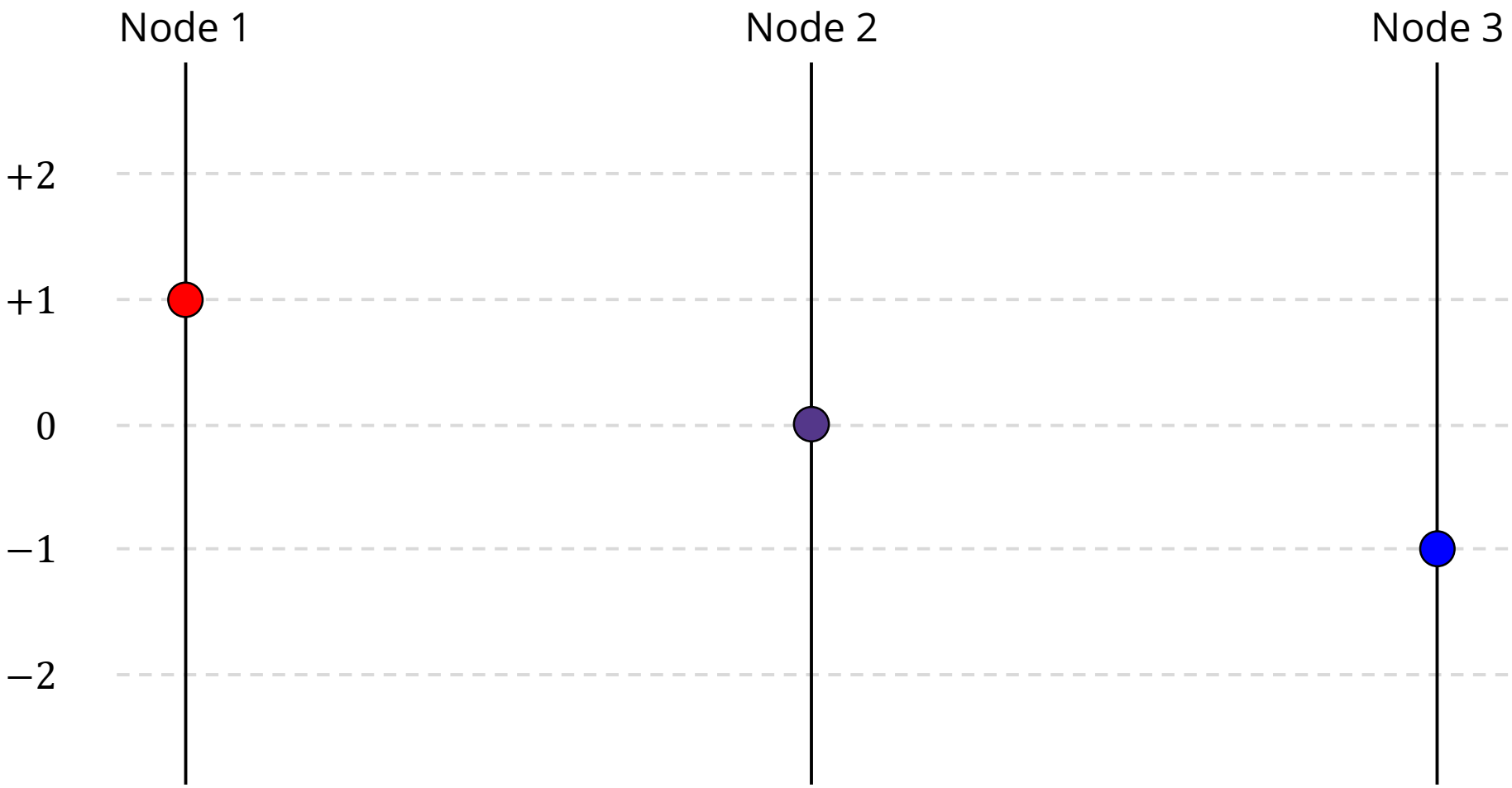

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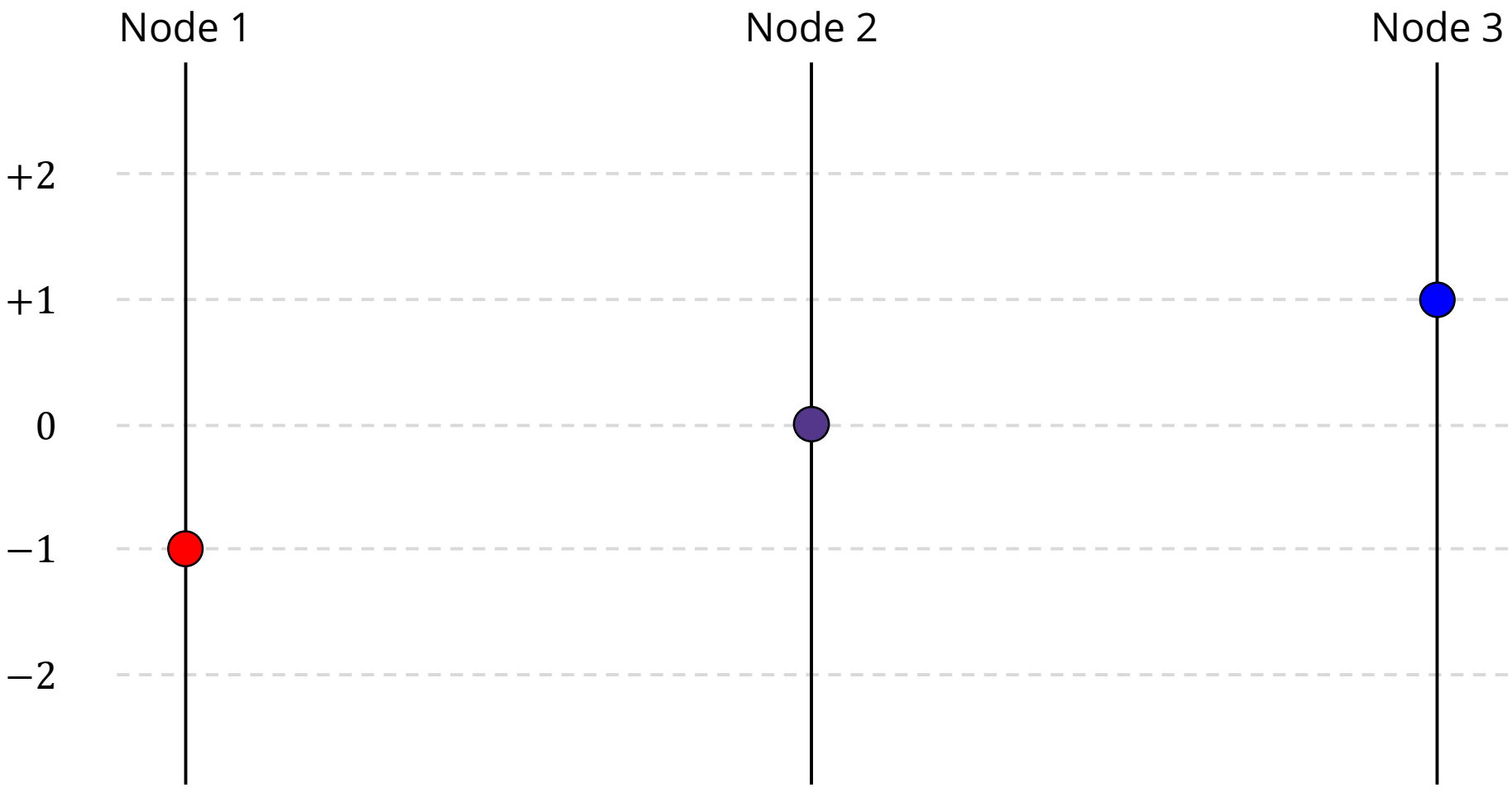
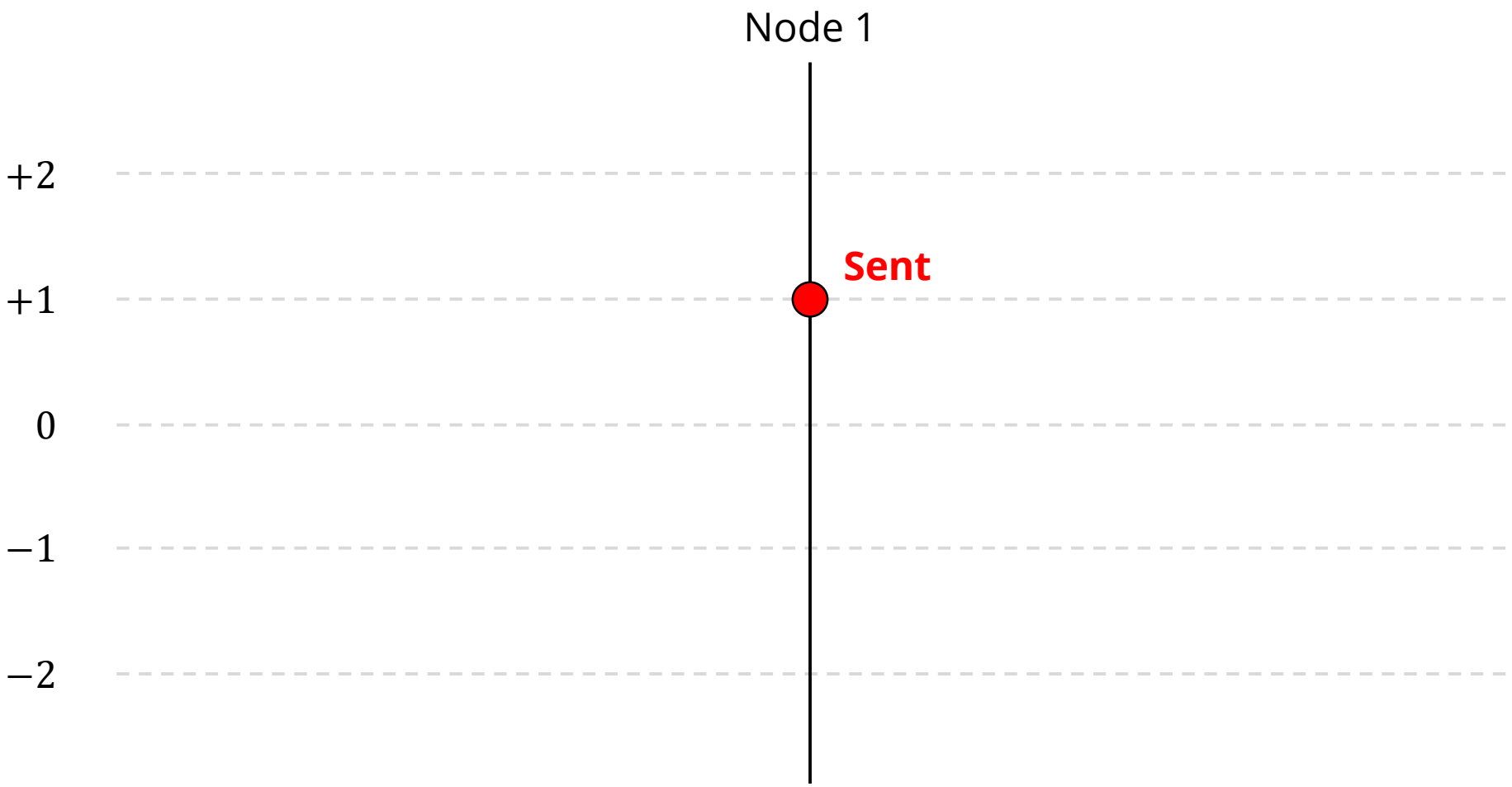


Analog Network Coding (ANC) – BPSK Example (2nd Time Slot)



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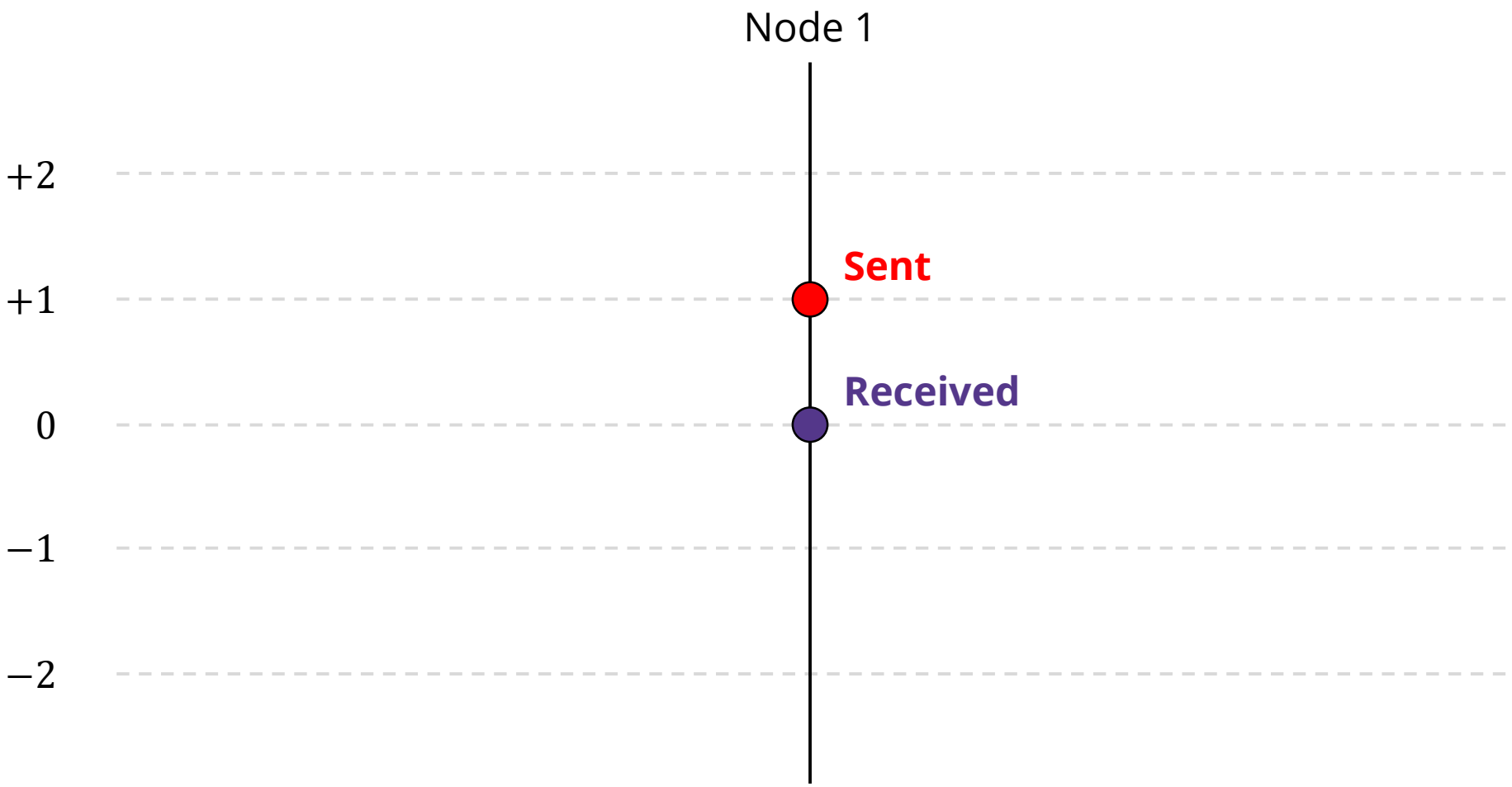

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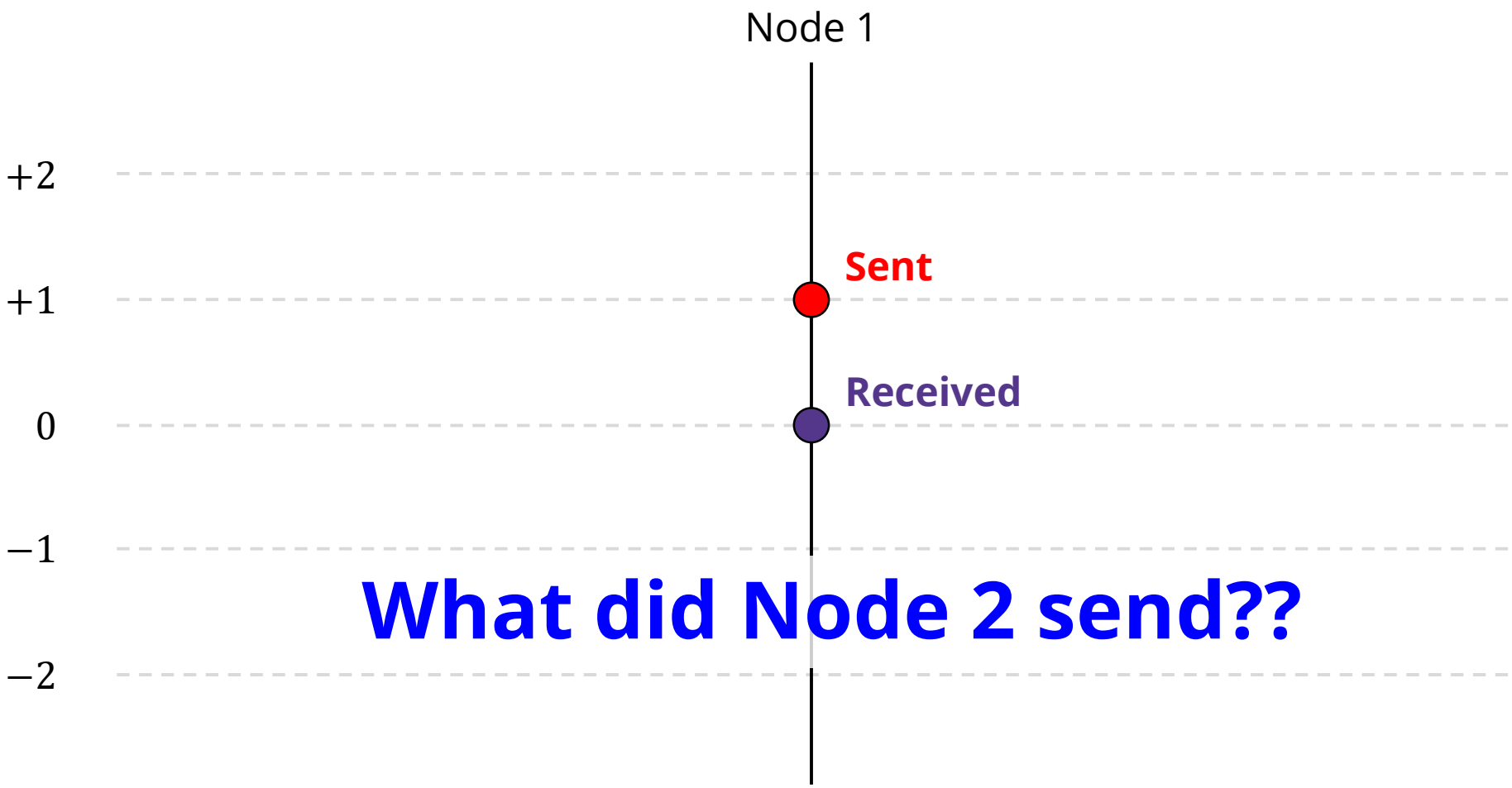

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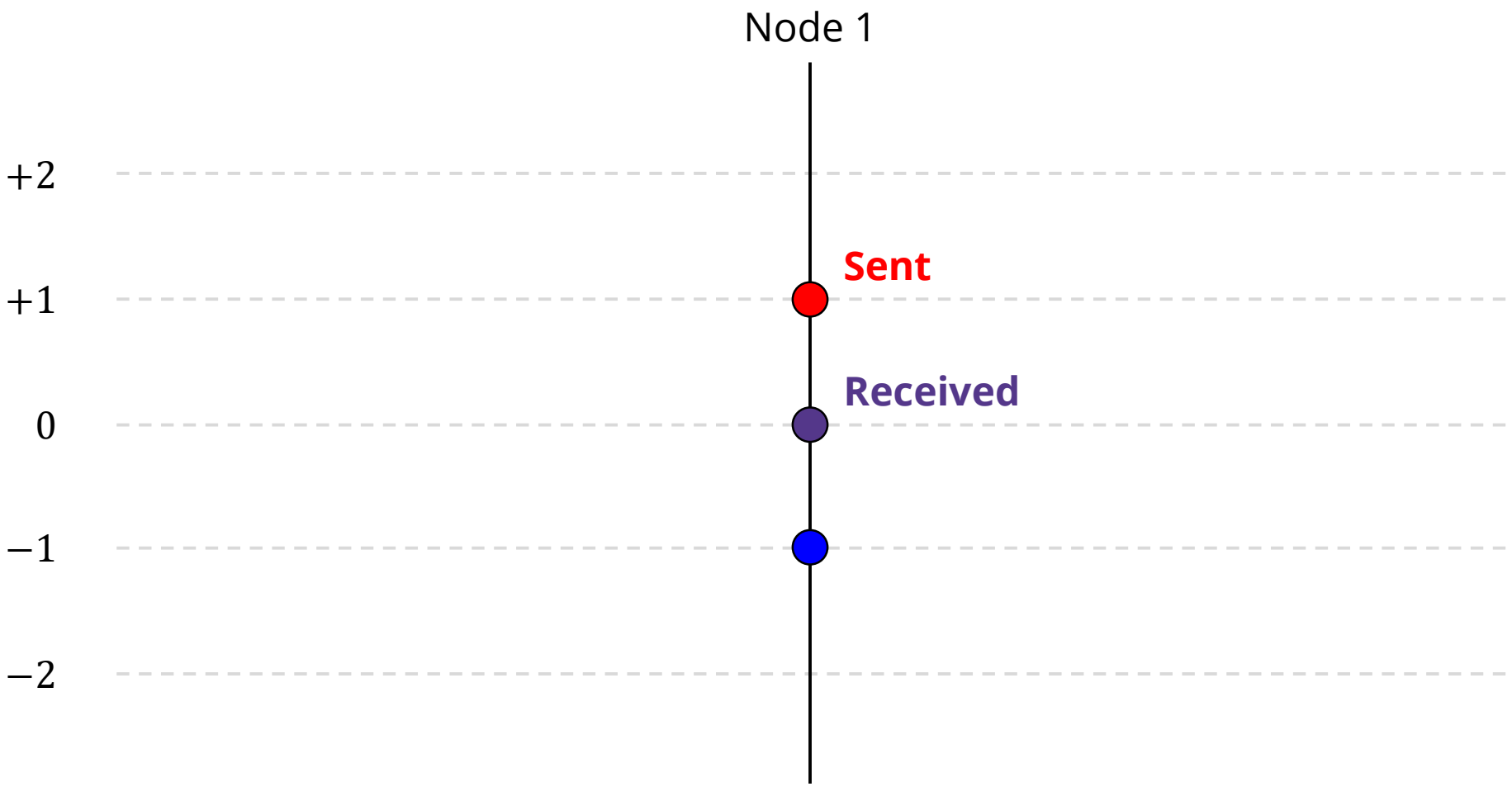

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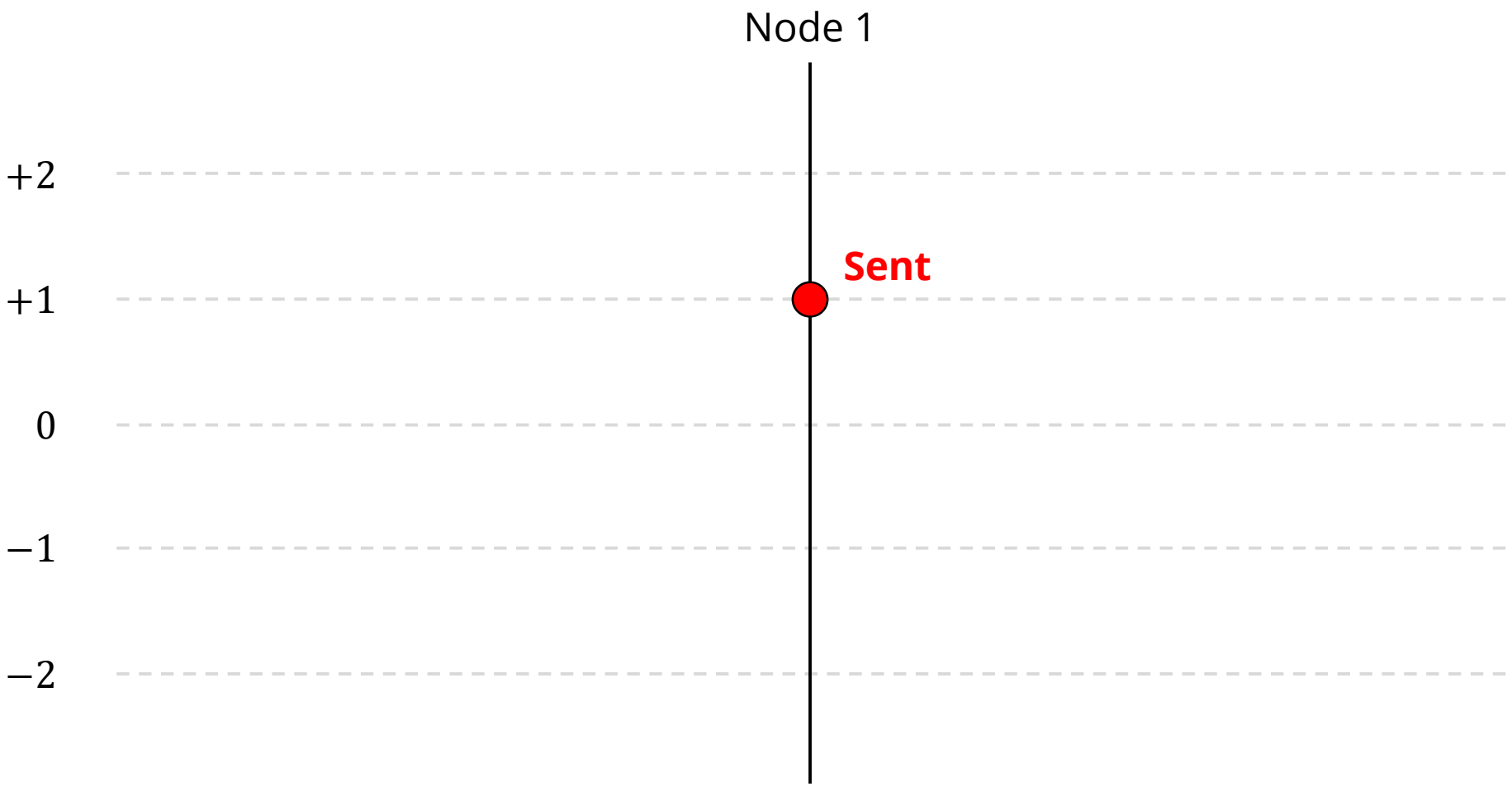

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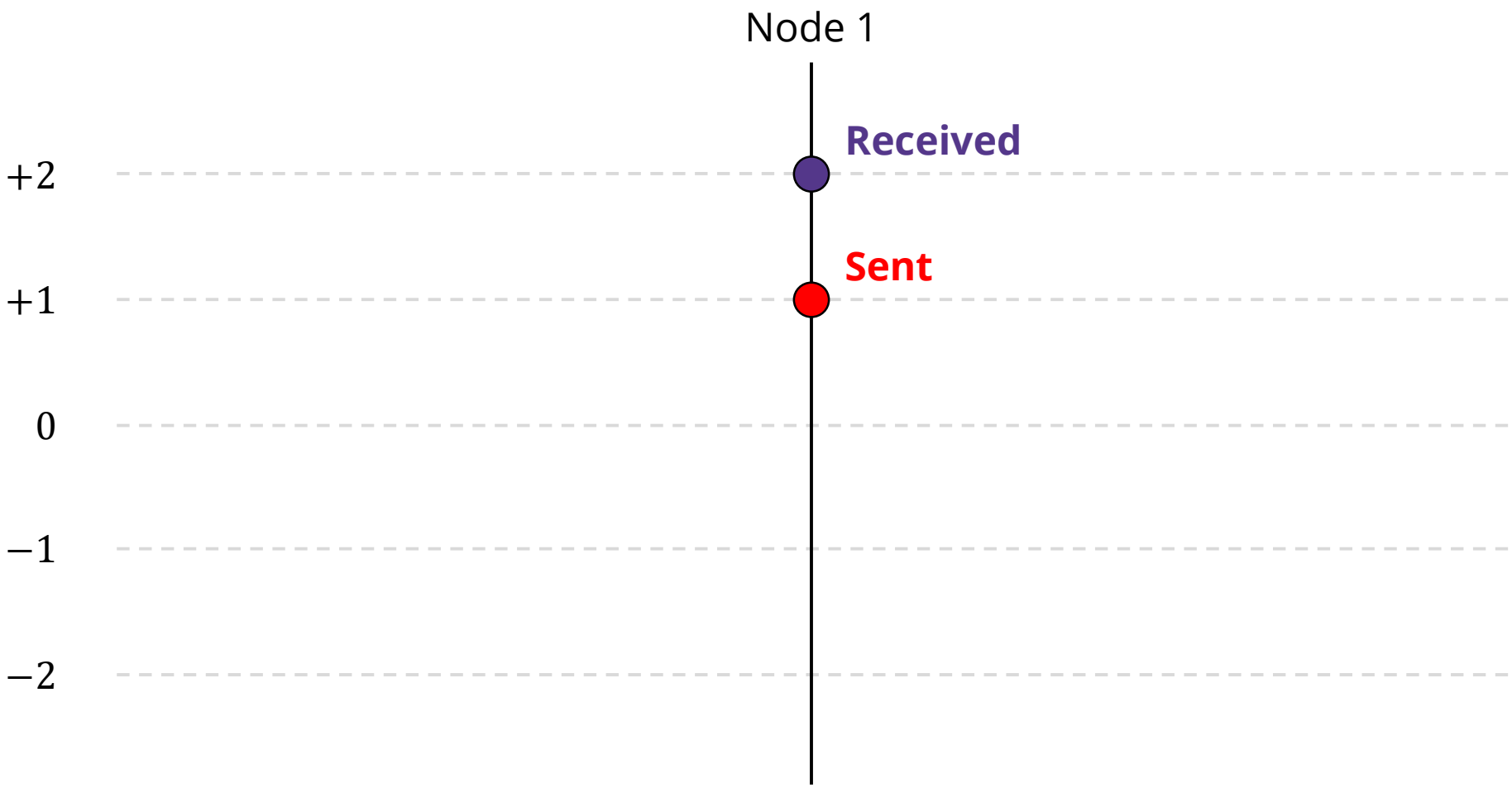

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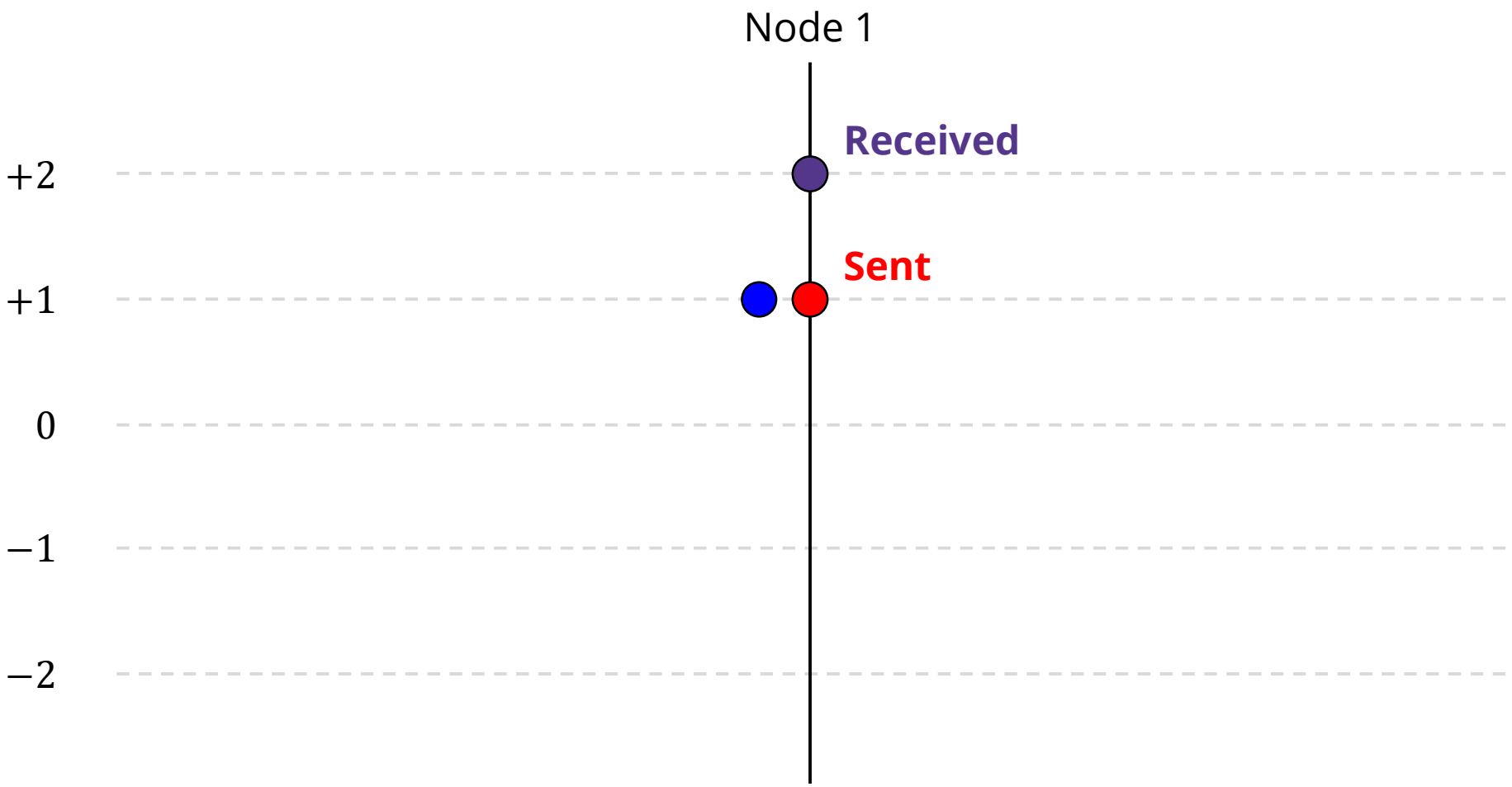

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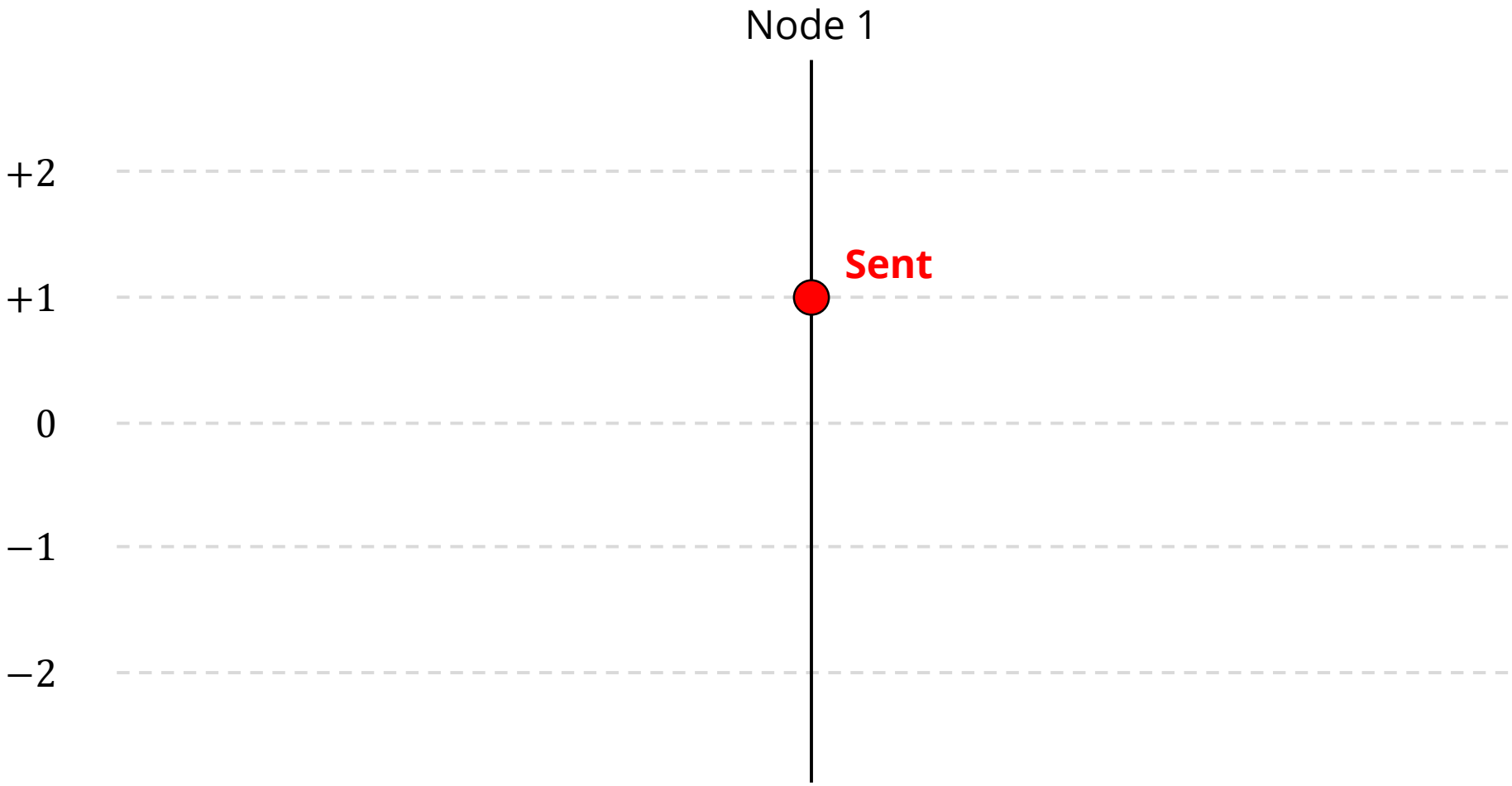

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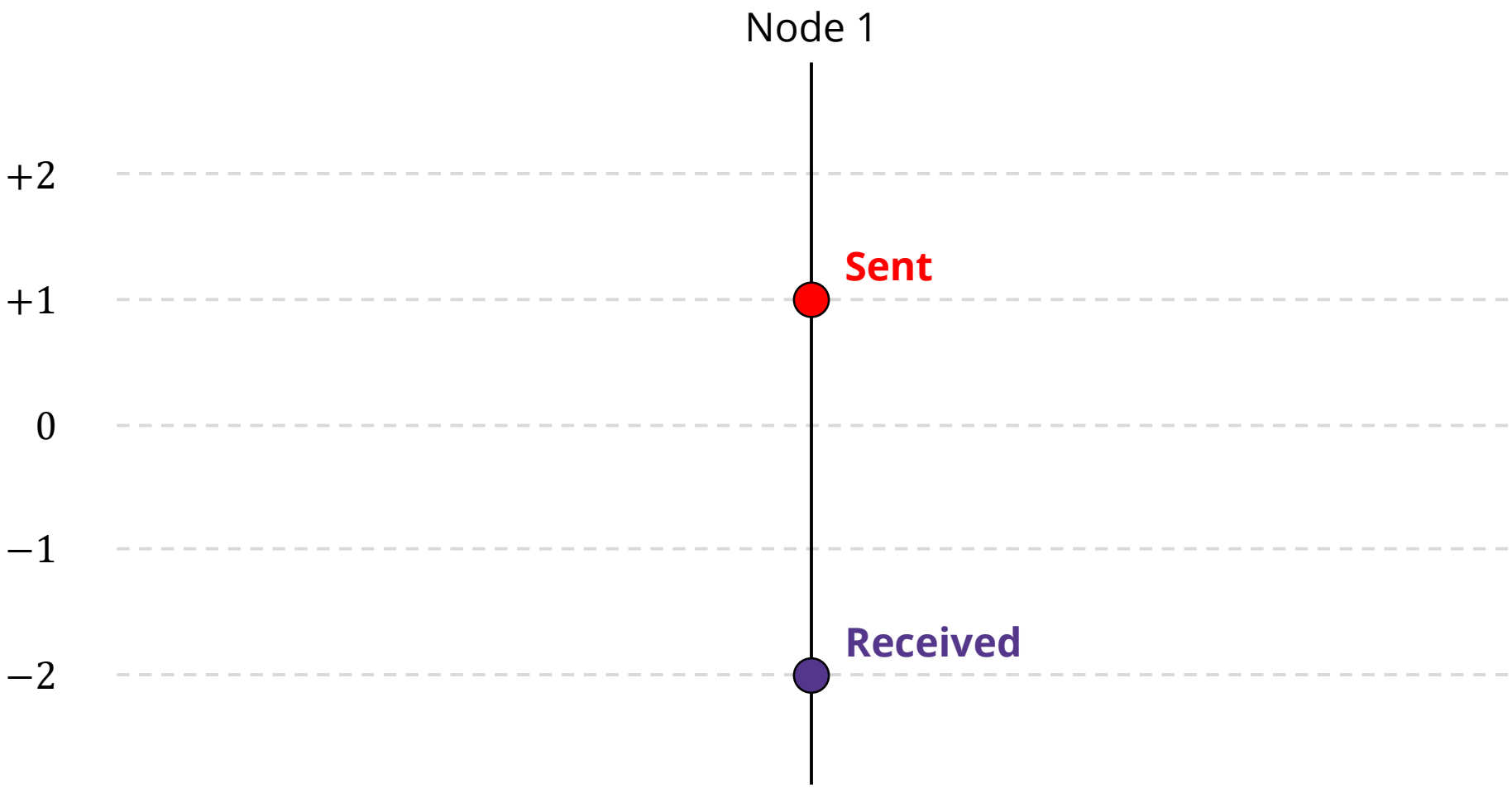

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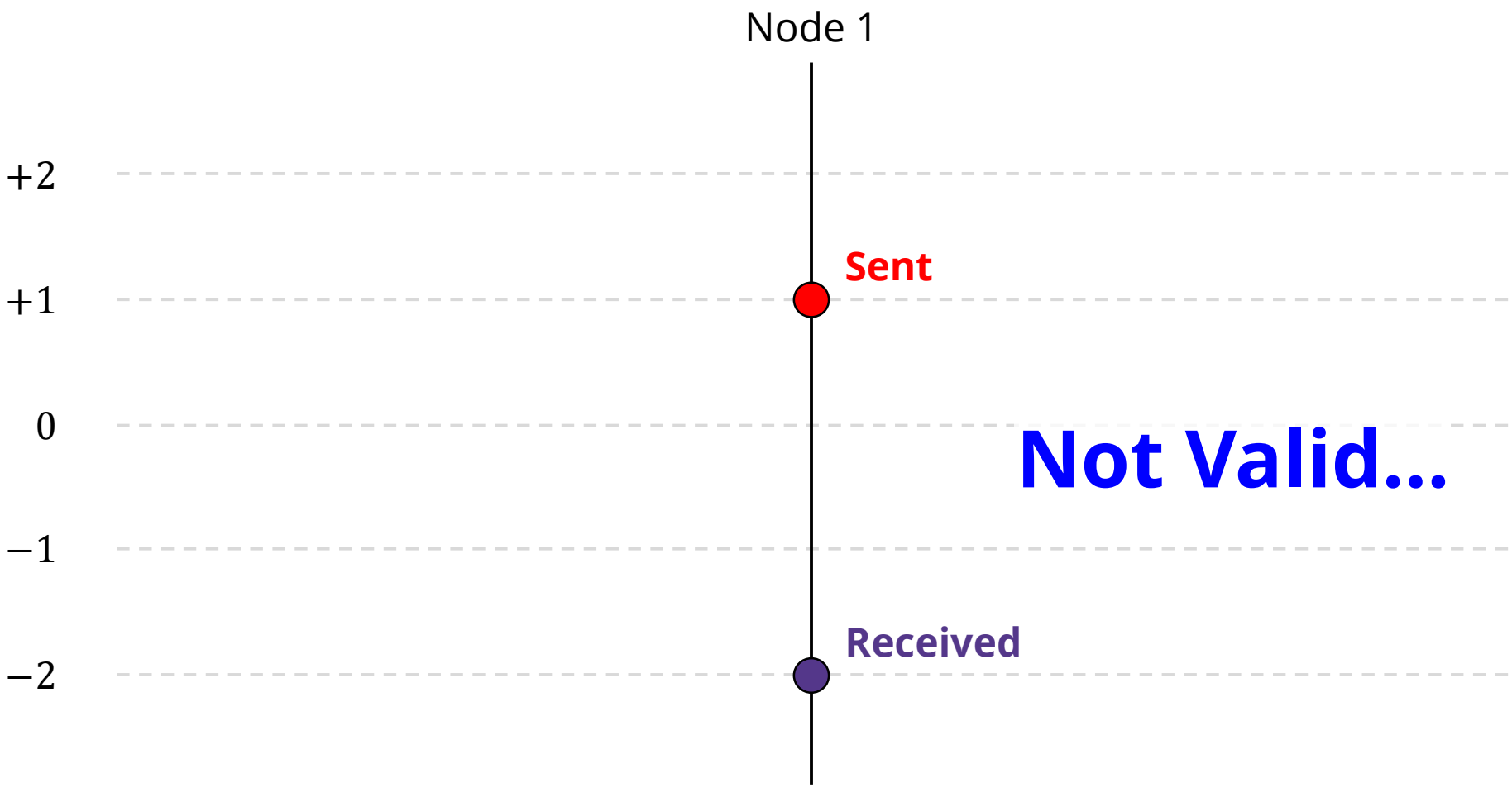


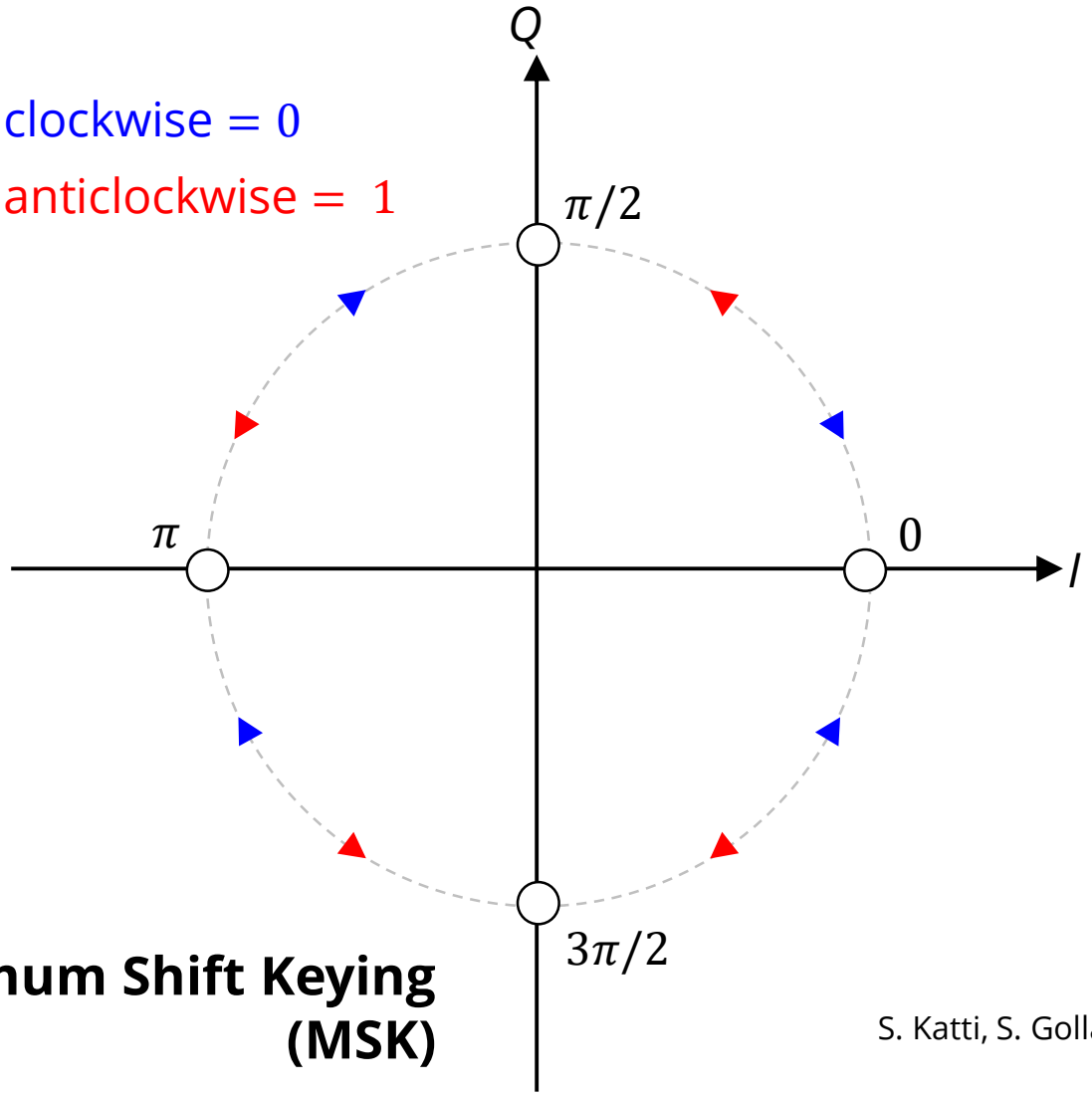
Analog Network Coding (ANC) – BPSK Example

- **What did this example assume?**
  1. No fading, i.e., no distortion of amplitude and phase
  2. Perfect synchronization
  3. Perfect detection of a collision
  4. Perfect knowledge of symbol used for decoding at each destination node

Analog Network Coding (ANC) – More Practical Example

$\pi/2$ clockwise $= 0$

$\pi/2$ anticlockwise $= 1$



Minimum Shift Keying (MSK)

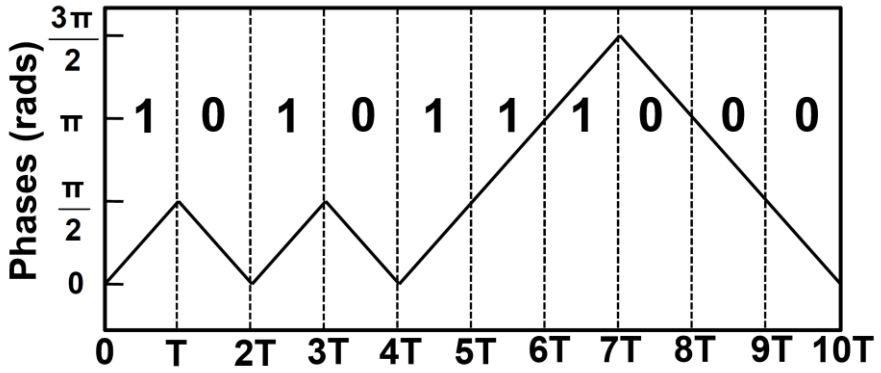


Figure 3—Example MSK modulation. MSK represents a "1" bit of as a phase difference of $\pi/2$ over for an interval T. It represents a a "0" bit as a phase difference of $-\pi/2$ over T.

S. Katti, S. Gollakota, and D. Katabi, "Embracing wireless Interference: Analog network coding," in *Proc. ACM SIGCOMM Computer Communication Review*, Aug. 2007.

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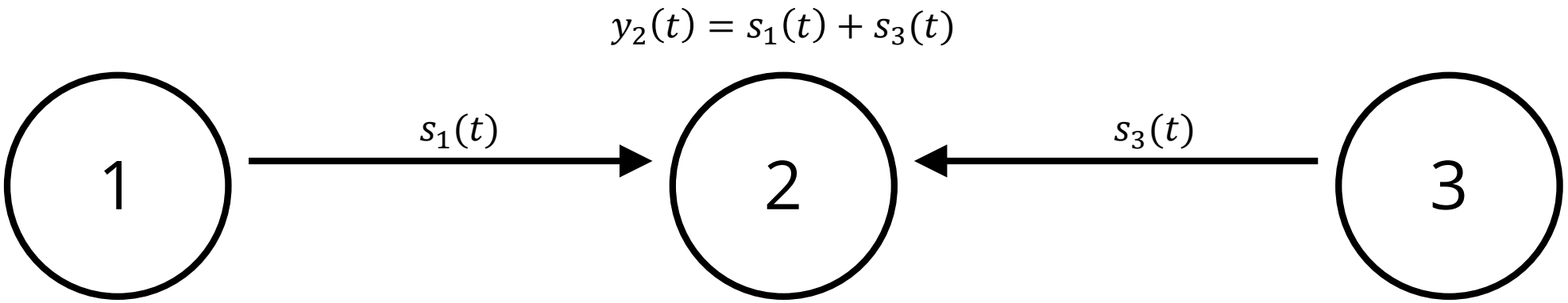

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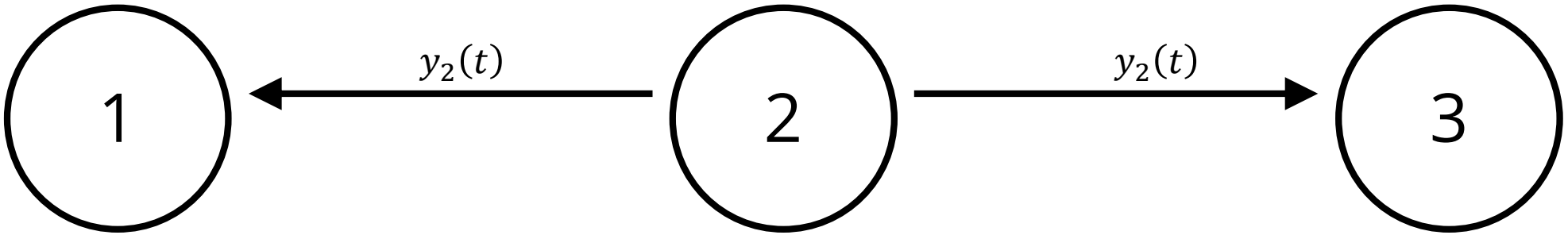
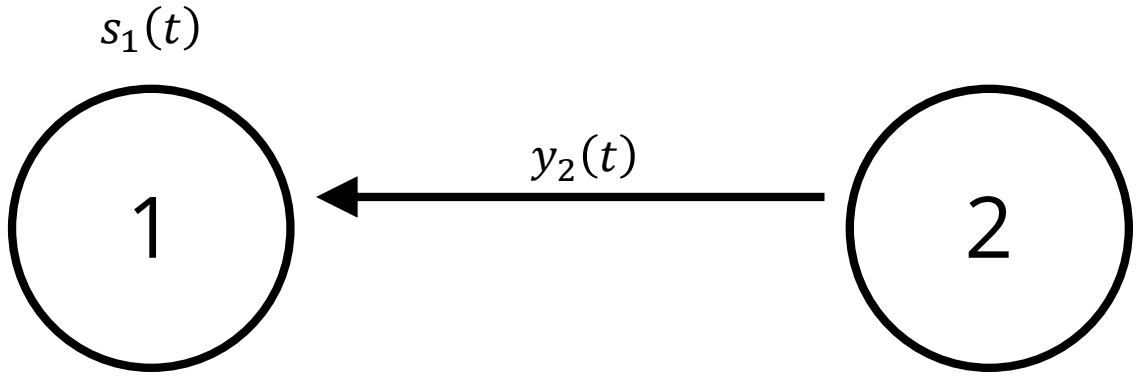


Analog Network Coding (ANC) – More Practical Example

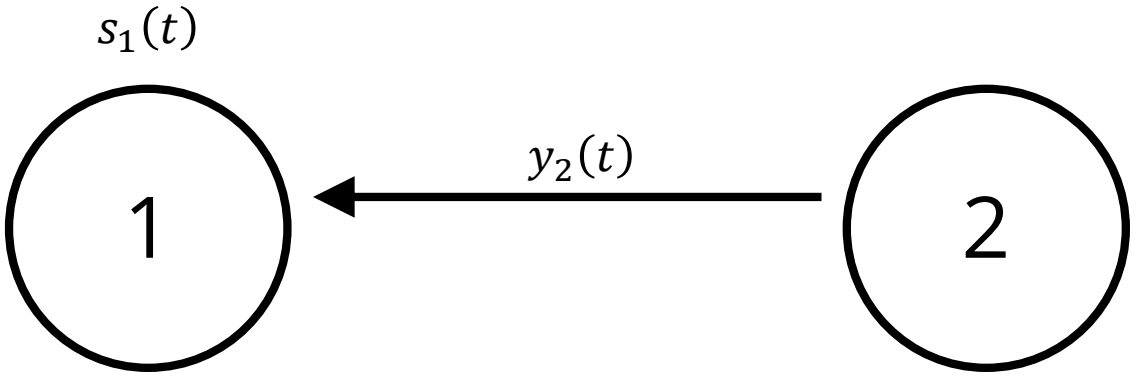
$$y_2(t) = h_1 s_1(t) e^{i(\theta_1(t)+\gamma_1)} + h_3 s_3(t) e^{i(\theta_3(t)+\gamma_3)}$$



Analog Network Coding (ANC) – More Practical Example

If, we can know these values…

$$y_2(t) = h_1 s_1(t) e^{i(\theta_1(t)+\gamma_1)} + h_2 s_2(t) e^{i(\theta_2(t)+\gamma_2)}$$



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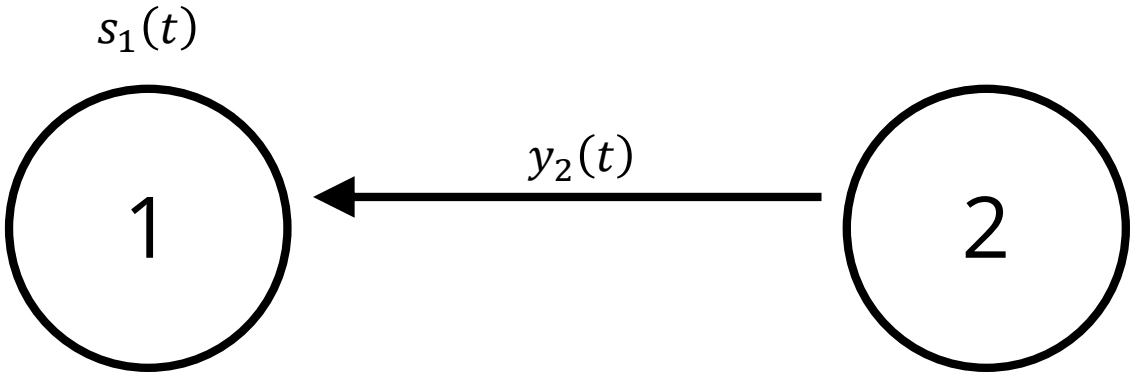

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$$y_2(t) = h_1 s_1(t) e^{i(\theta_1(t)+\gamma_1)} + h_2 s_2(t) e^{i(\theta_2(t)+\gamma_2)}$$

How?? In the case of the **phase**…

$$r = \frac{h_1 s_1(t+1) e^{i(\theta_1(t+1)+\gamma_1)}}{h_1 s_1(t) e^{i(\theta_1(t)+\gamma_1)}} = e^{i(\theta_1(t+1)-\theta_1(t))}$$

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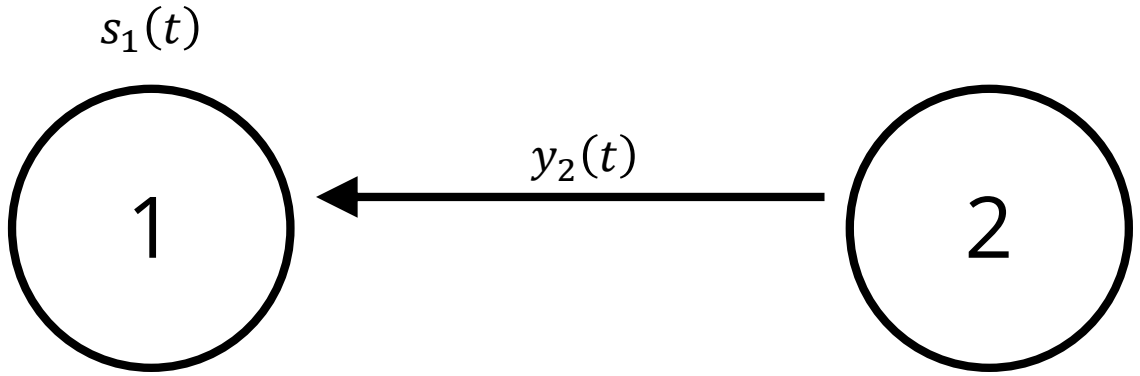

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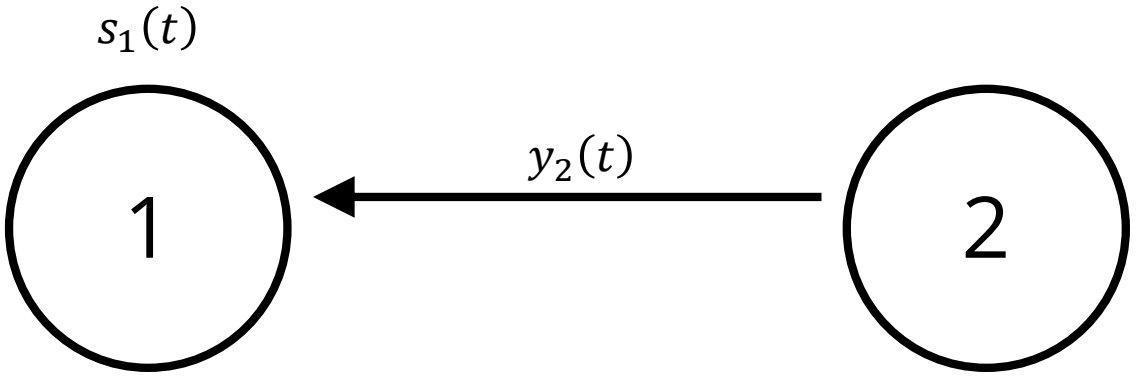
Since we calculate the difference in the phase, we can know what was transmitted before because of the characteristic of MSK

Analog Network Coding (ANC) – More Practical Example



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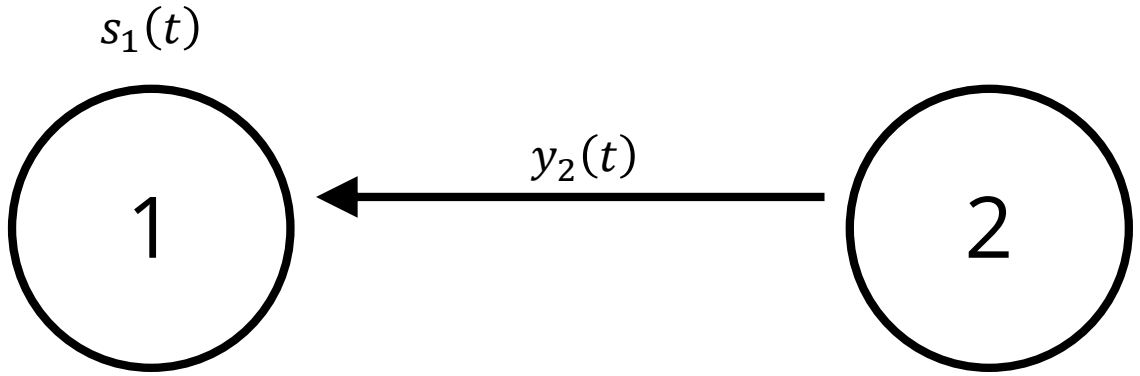
$y = hx$, where x denotes training signal

Thus, the estimated channel $\hat{h} = y/x$

Analog Network Coding (ANC) – More Practical Example



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How?? In the case of the **channel**…

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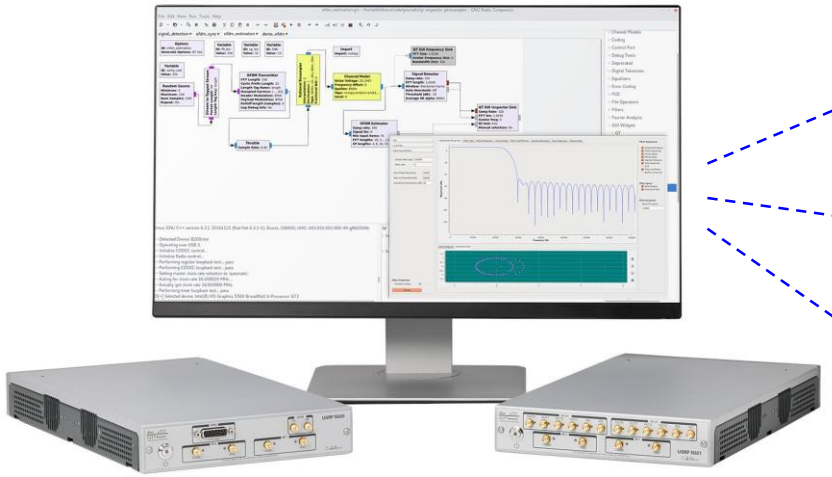
Thus, the estimated channel $\hat{h} = y/x$

This simple method is called least square (LS) channel estimation

Analog Network Coding (ANC) – Implementation using SDR



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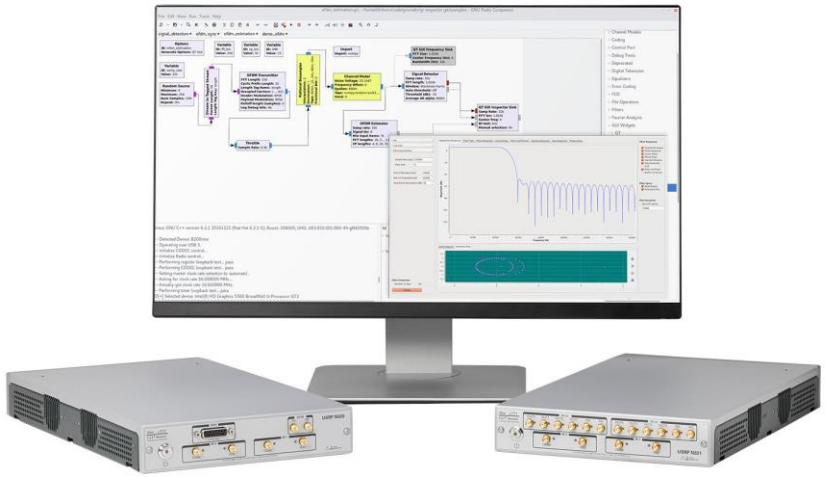
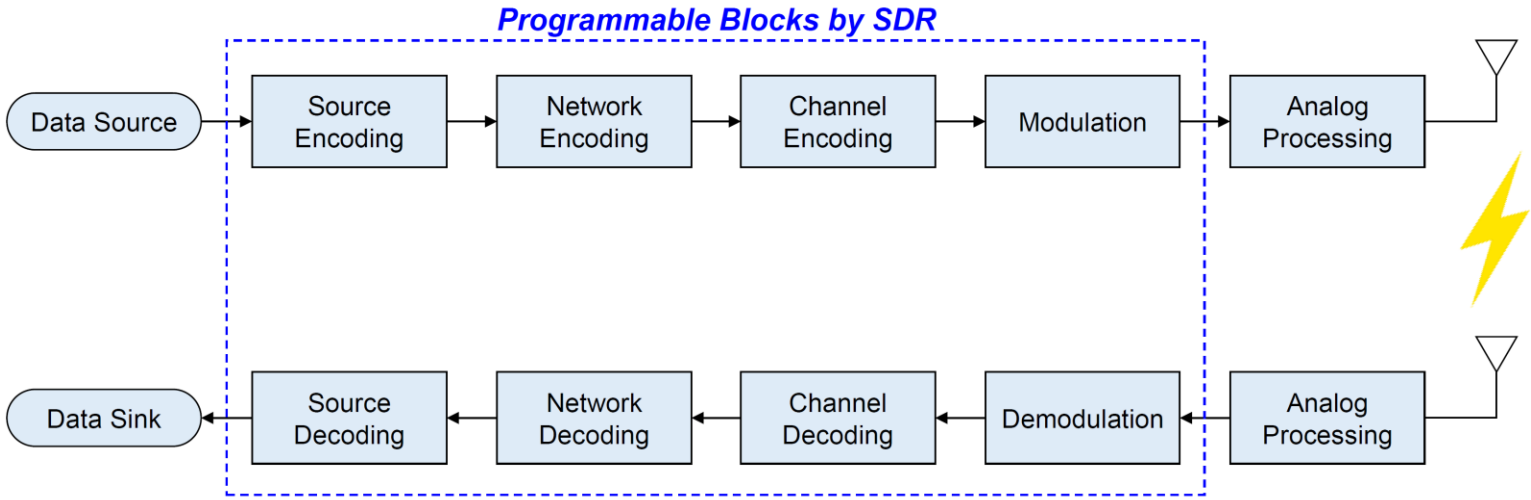


Ettus Research USRP N320



Analog Network Coding (ANC) – Implementation using SDR

- **ANC is relatively simpler than PNC to implement by Hardware**
- **However, easy to cause error propagation because of AF relaying**

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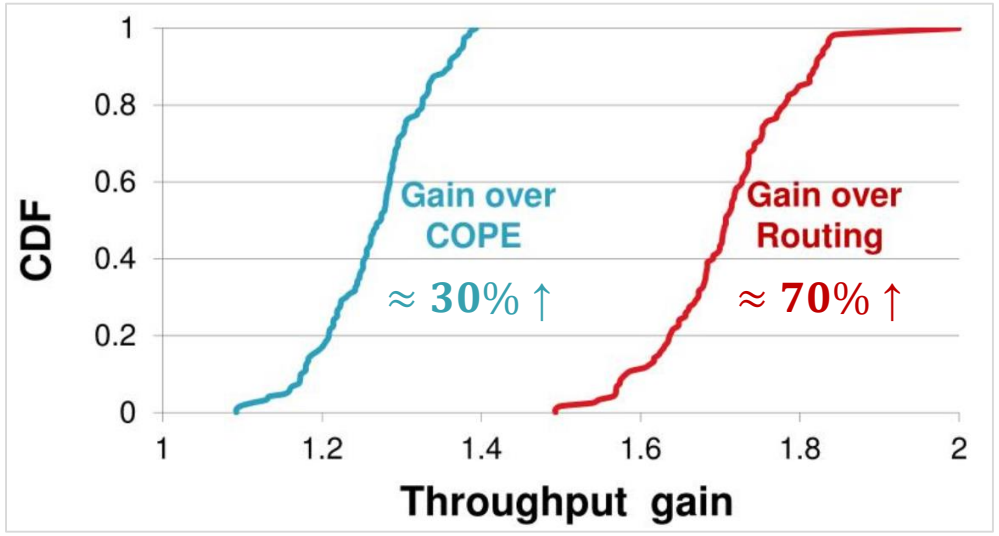

ANC

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- ANC is relatively simpler than PNC to imp
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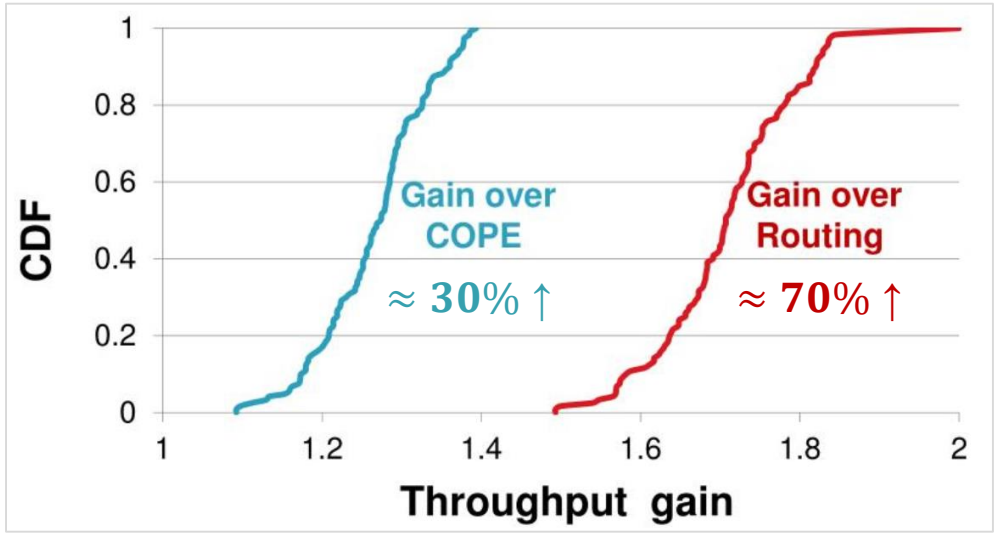
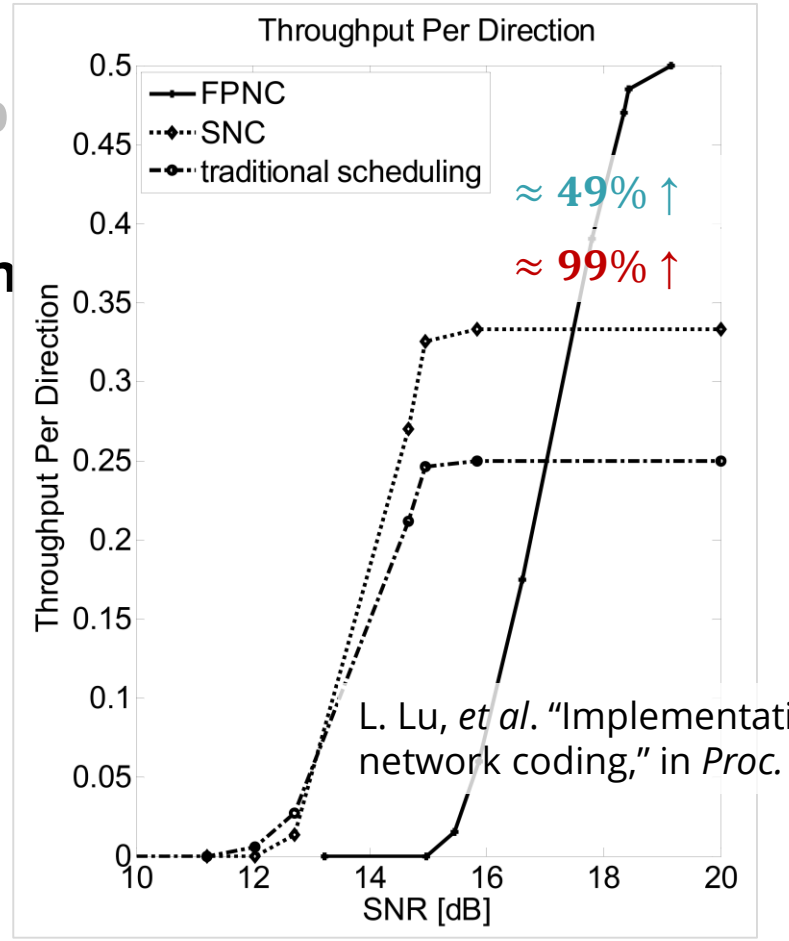


ANC



L. Lu, *et al*. "Implementation of physical-layer network coding," in *Proc. IEEE ICC*, 2012

PNC

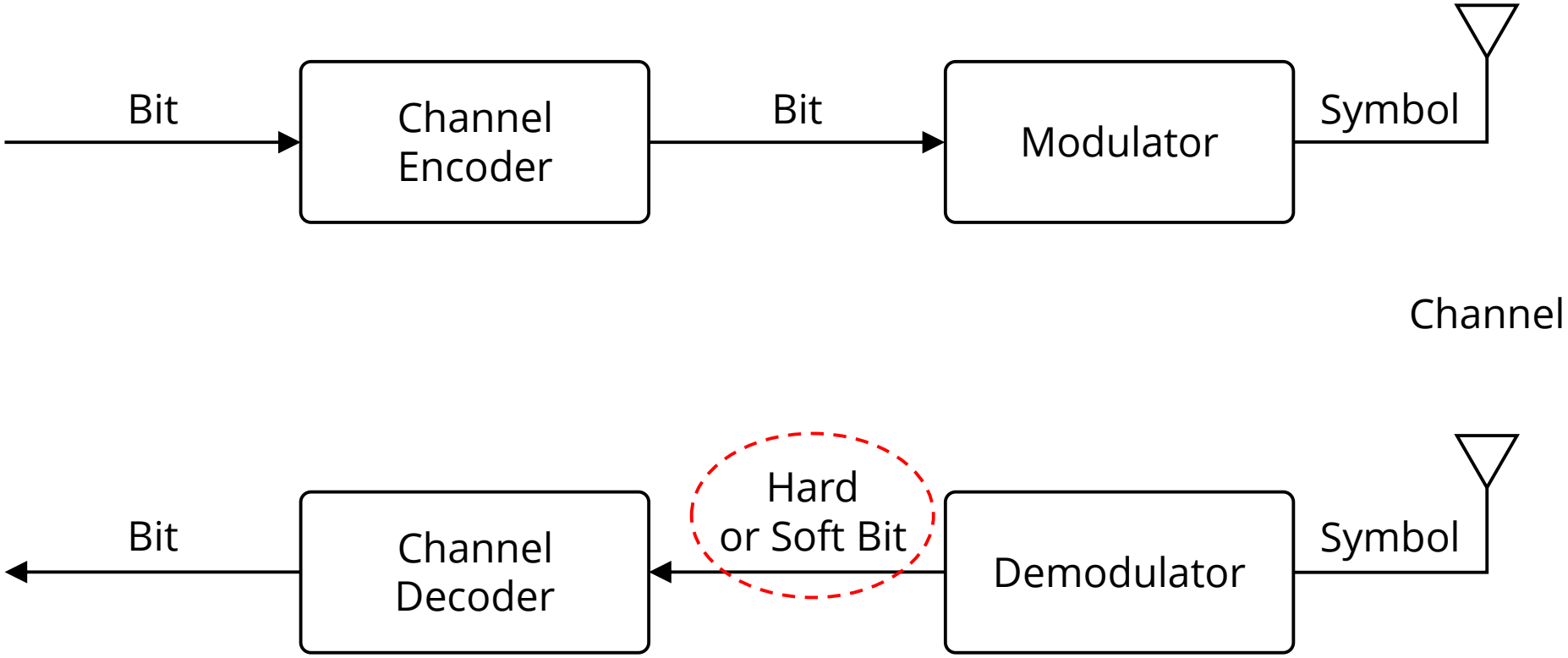
Analog Network Coding (ANC) – Implementation using SDR

- ANC is relatively simpler than PNC to implement by Hardware
- However, easy to cause error propagation because of AF relaying
- **In summary,**

	ANC (AF-like)	**PNC (DF-like)**
Complexity	Low	High
Throughput	Low	High

Soft Bit-based Network Coding

Soft Bit-based Network Coding – What is Soft Bit?



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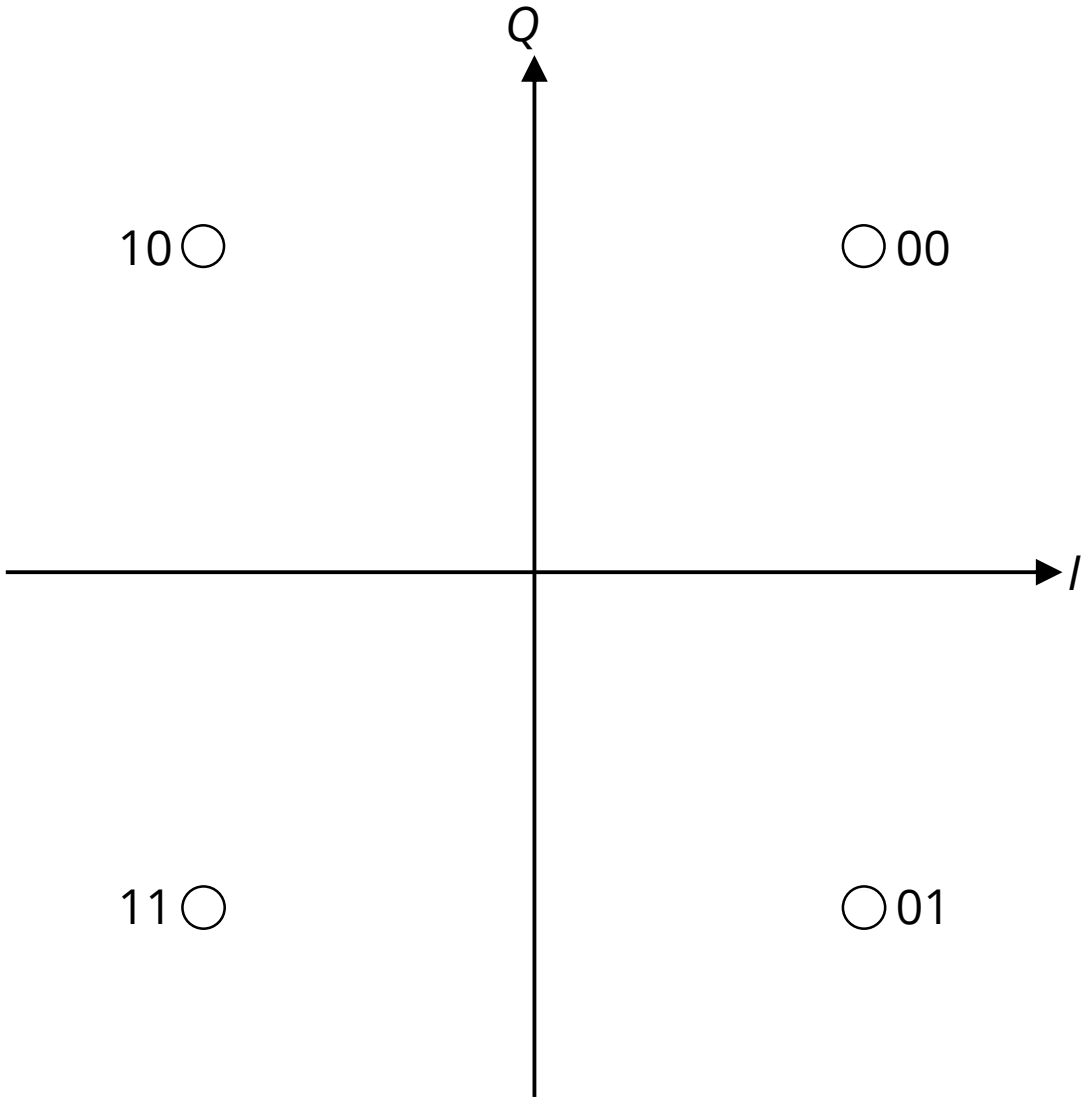
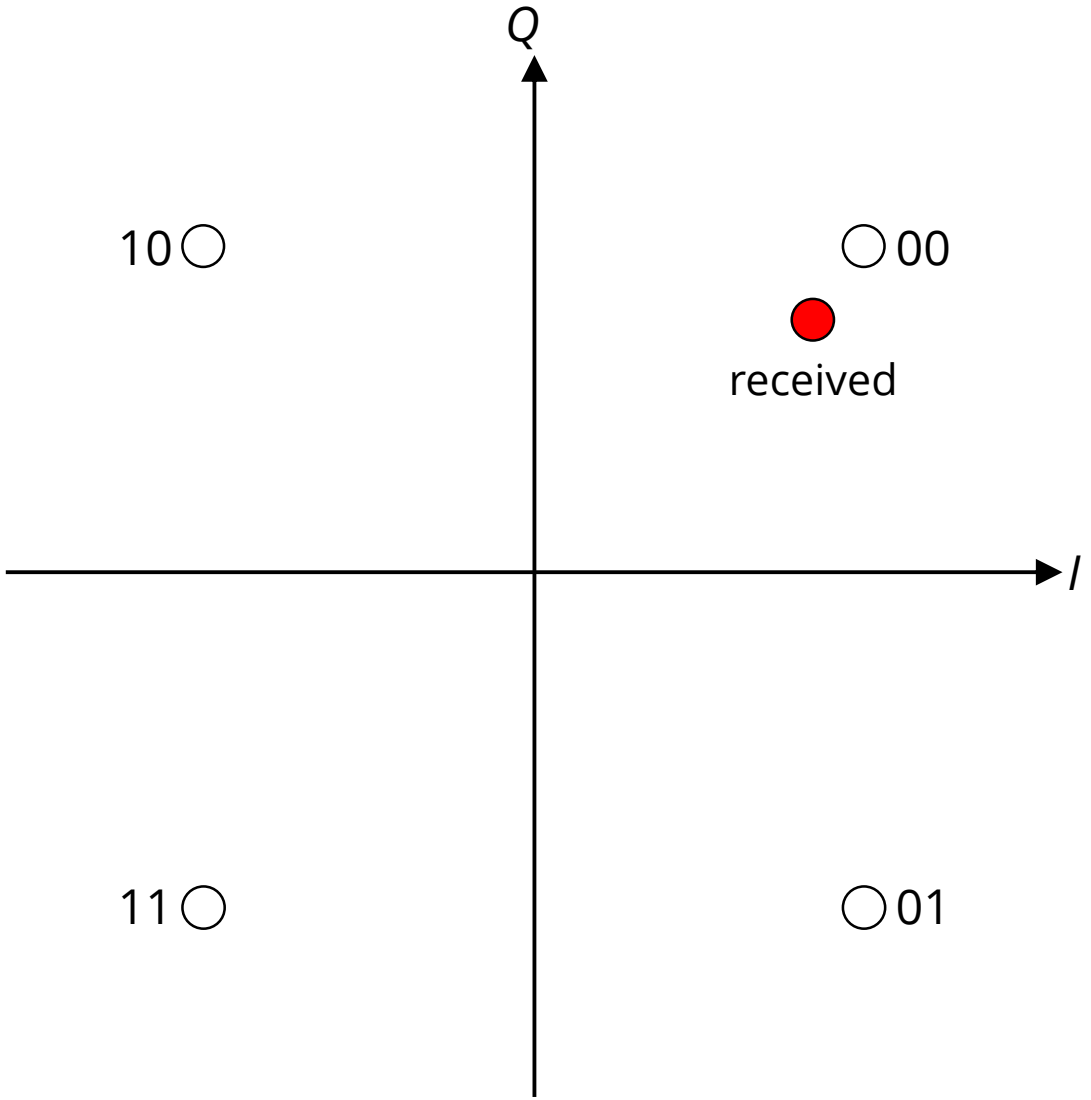


Soft Bit-based Network Coding – What is Soft Bit?

Case 1



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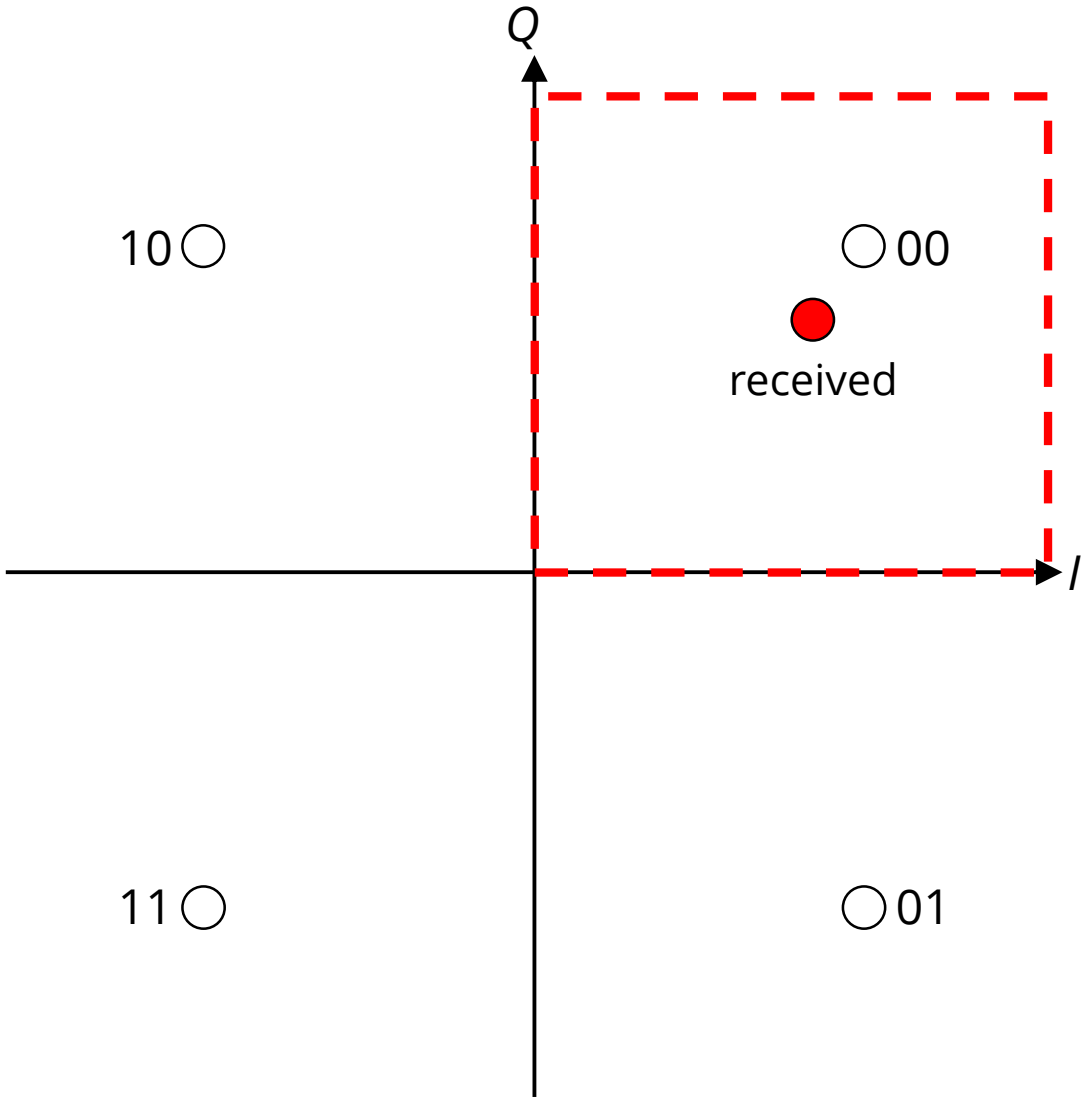

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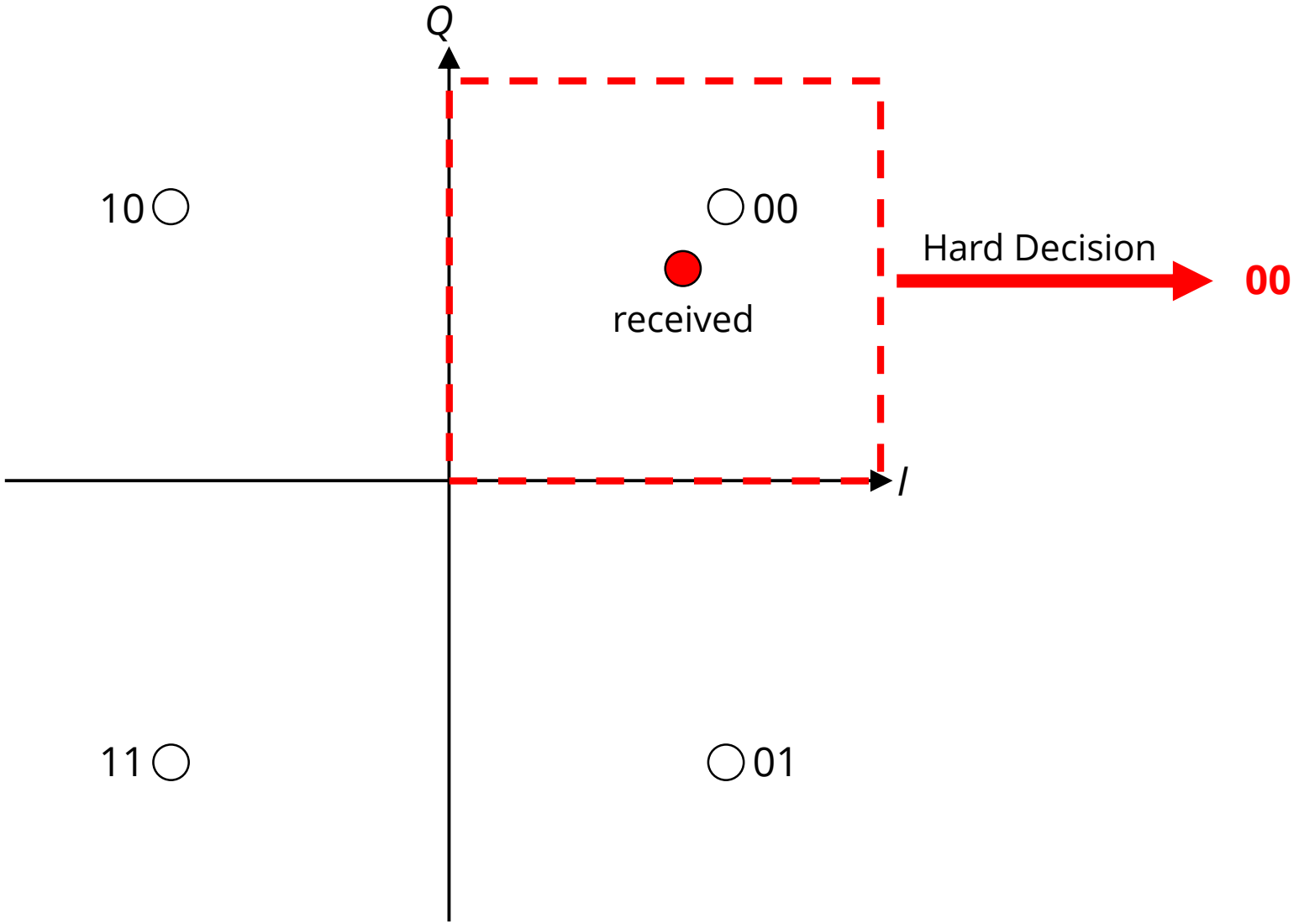
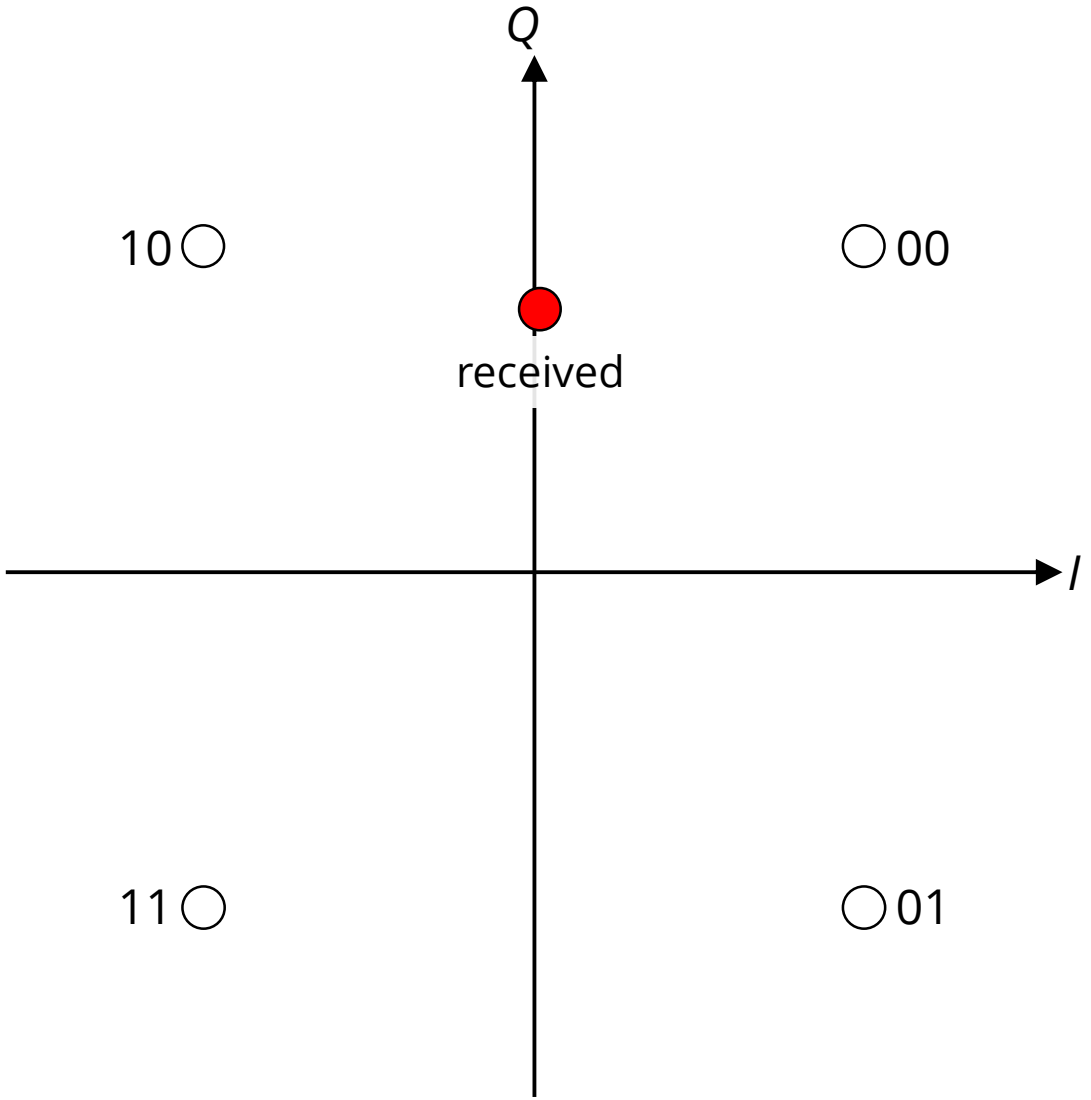


Soft Bit-based Network Coding – What is Soft Bit?

Case 2



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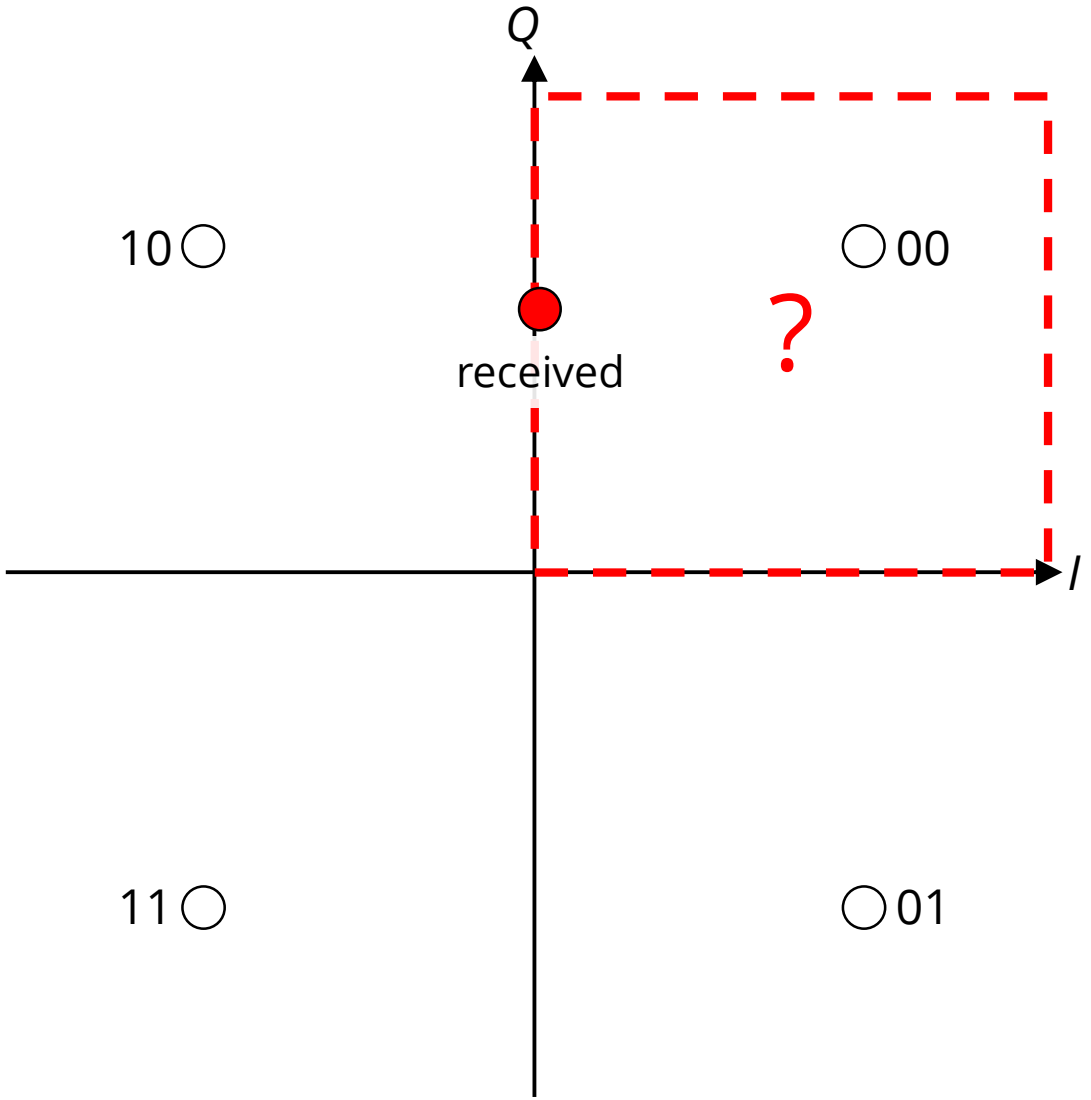

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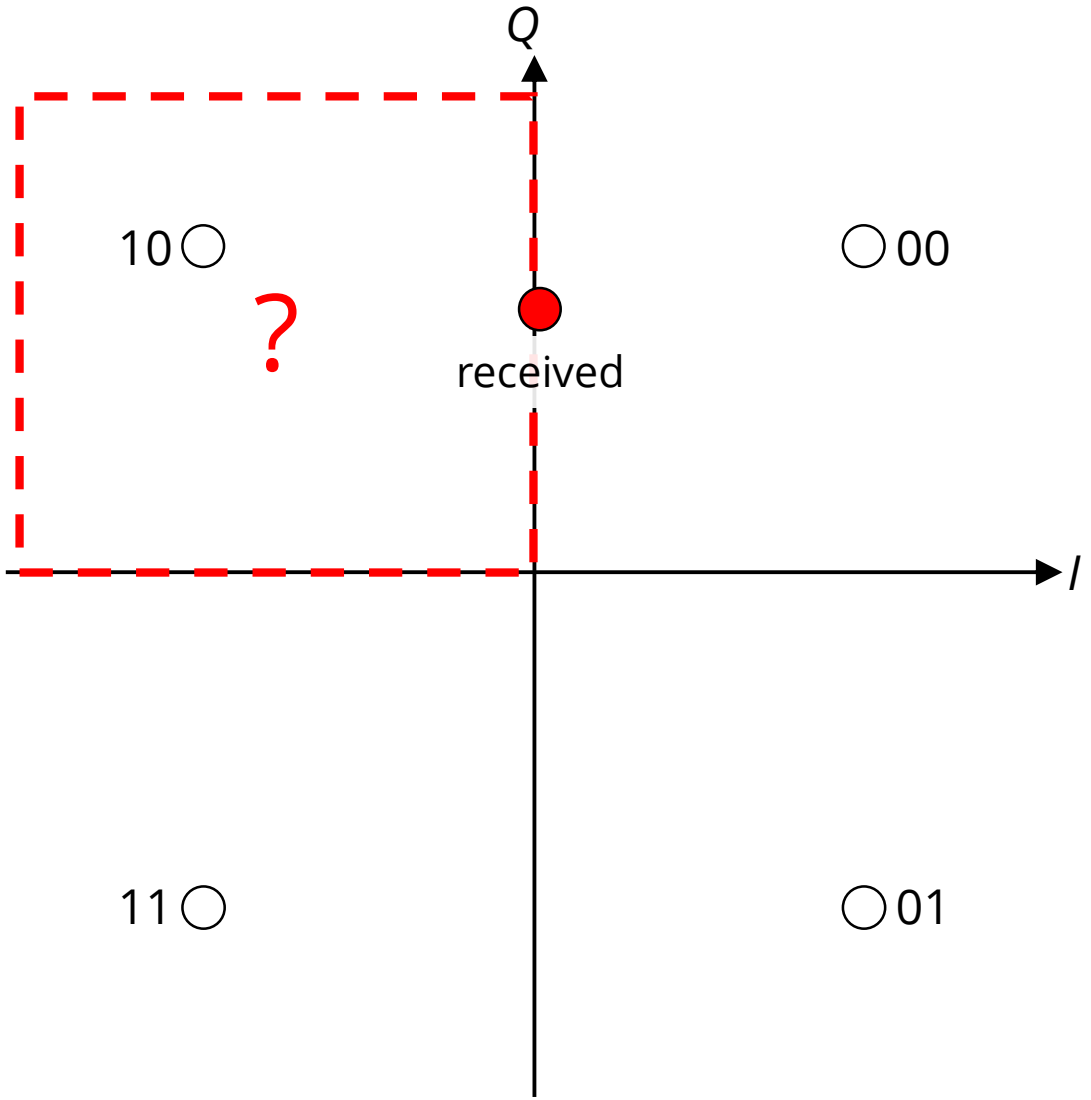

Soft Bit-based Network Coding – What is Soft Bit?

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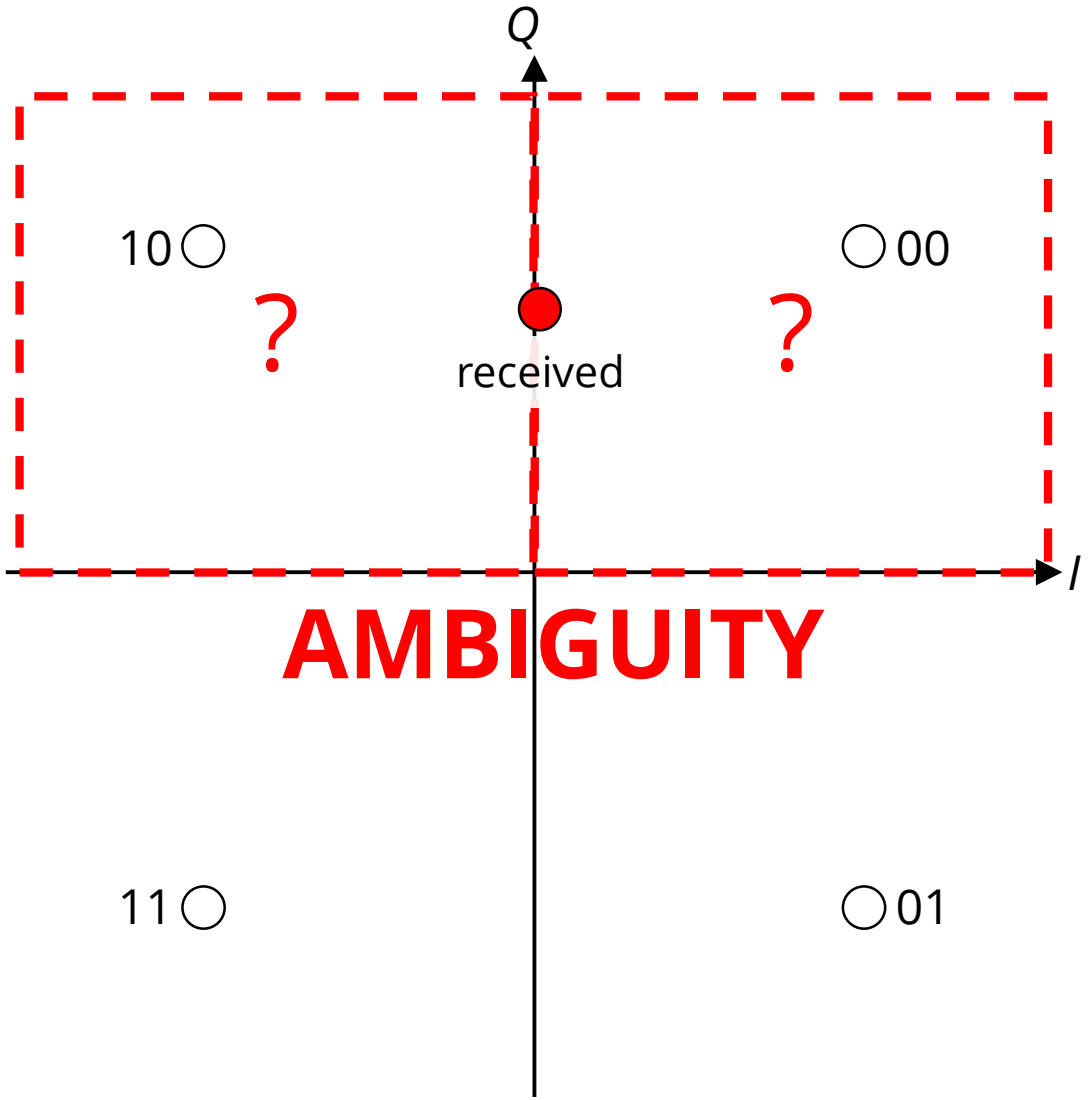

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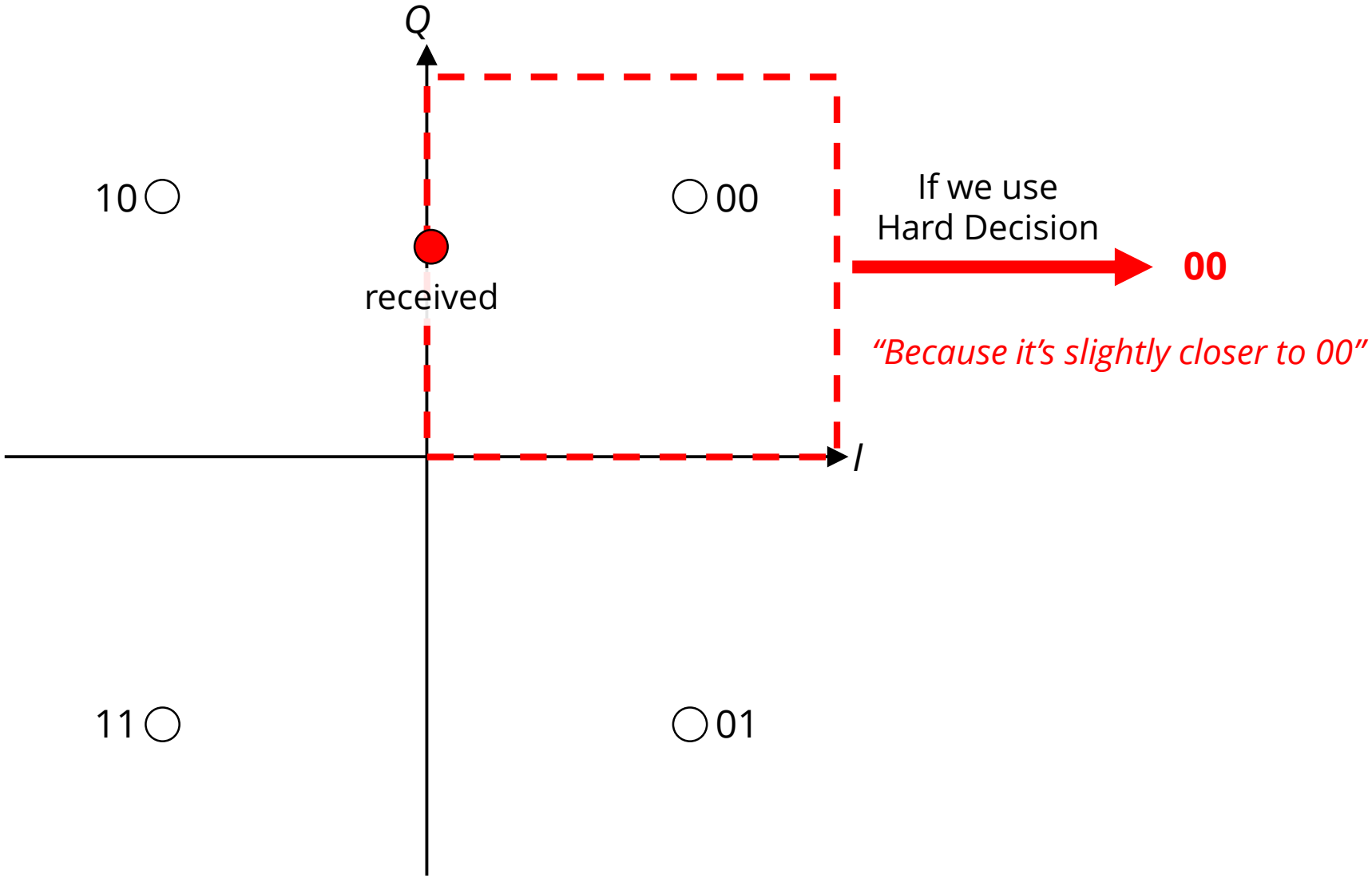

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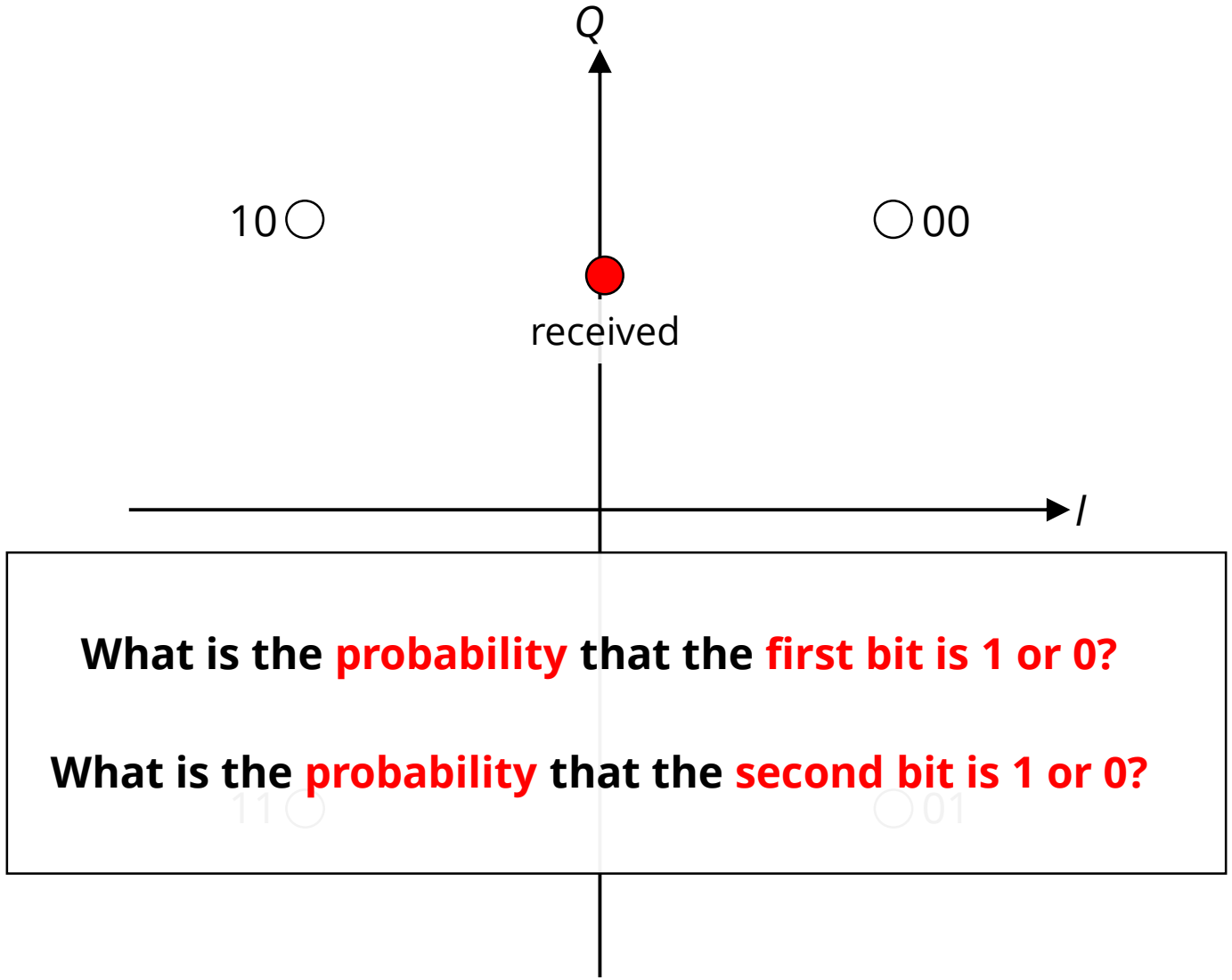

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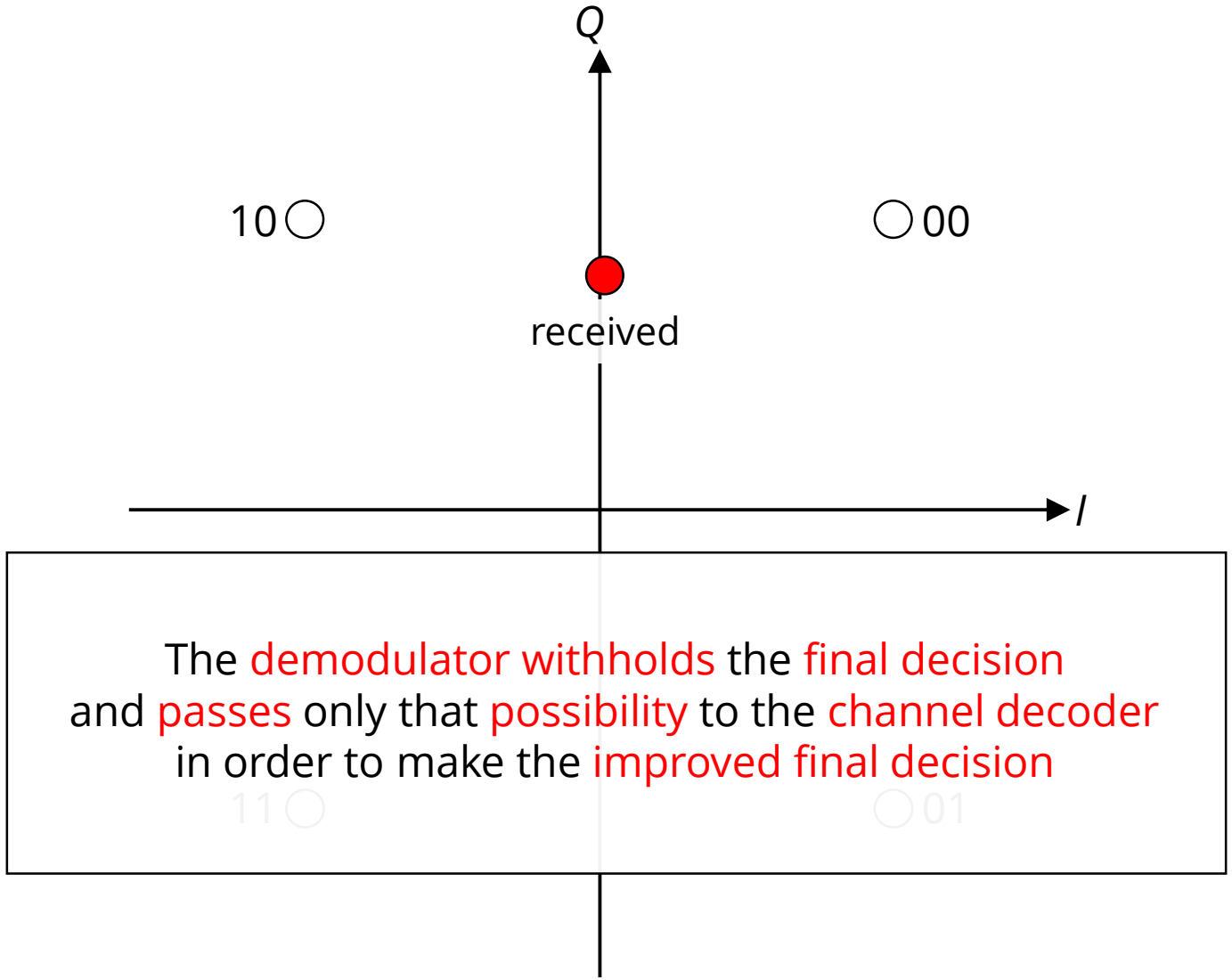

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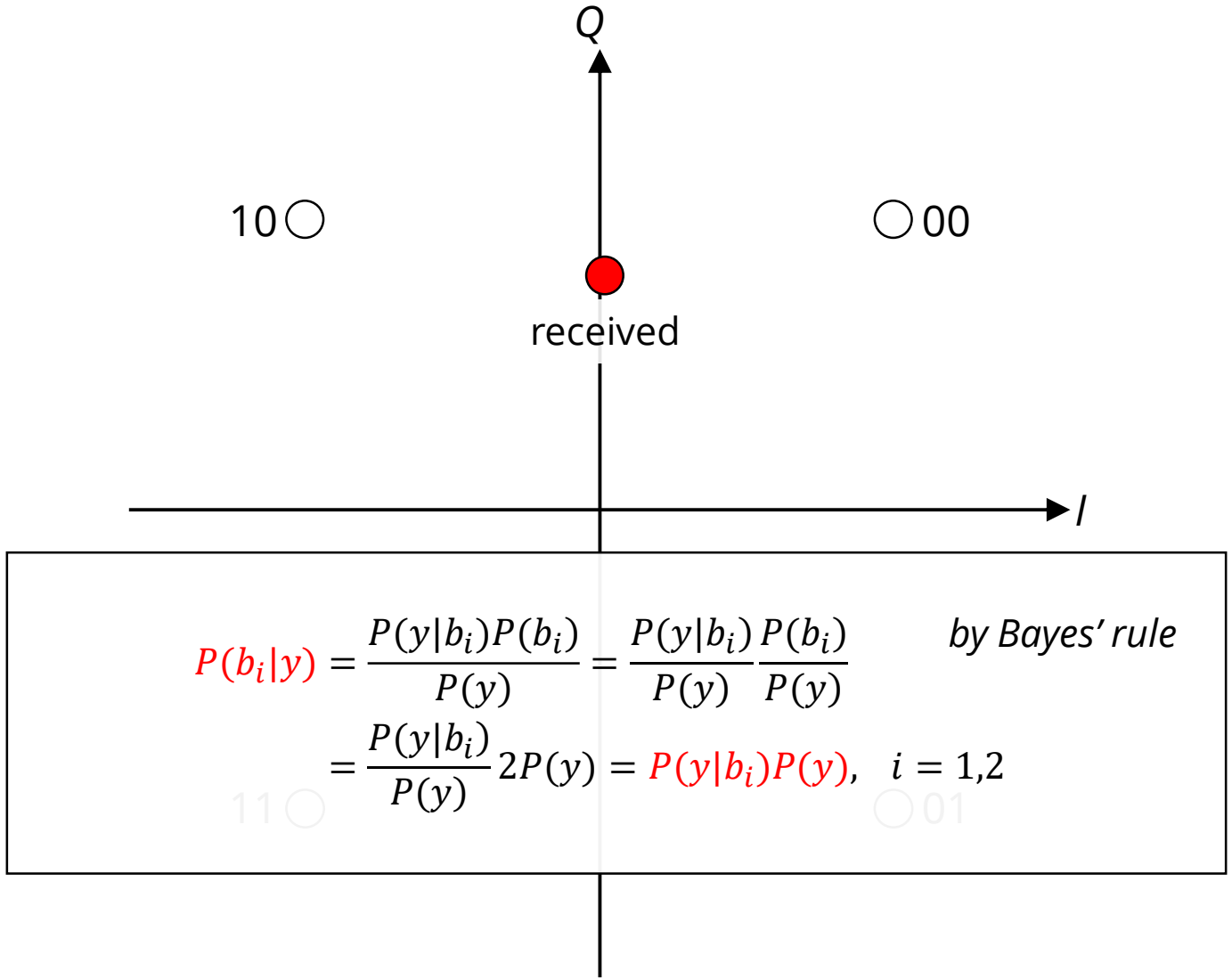
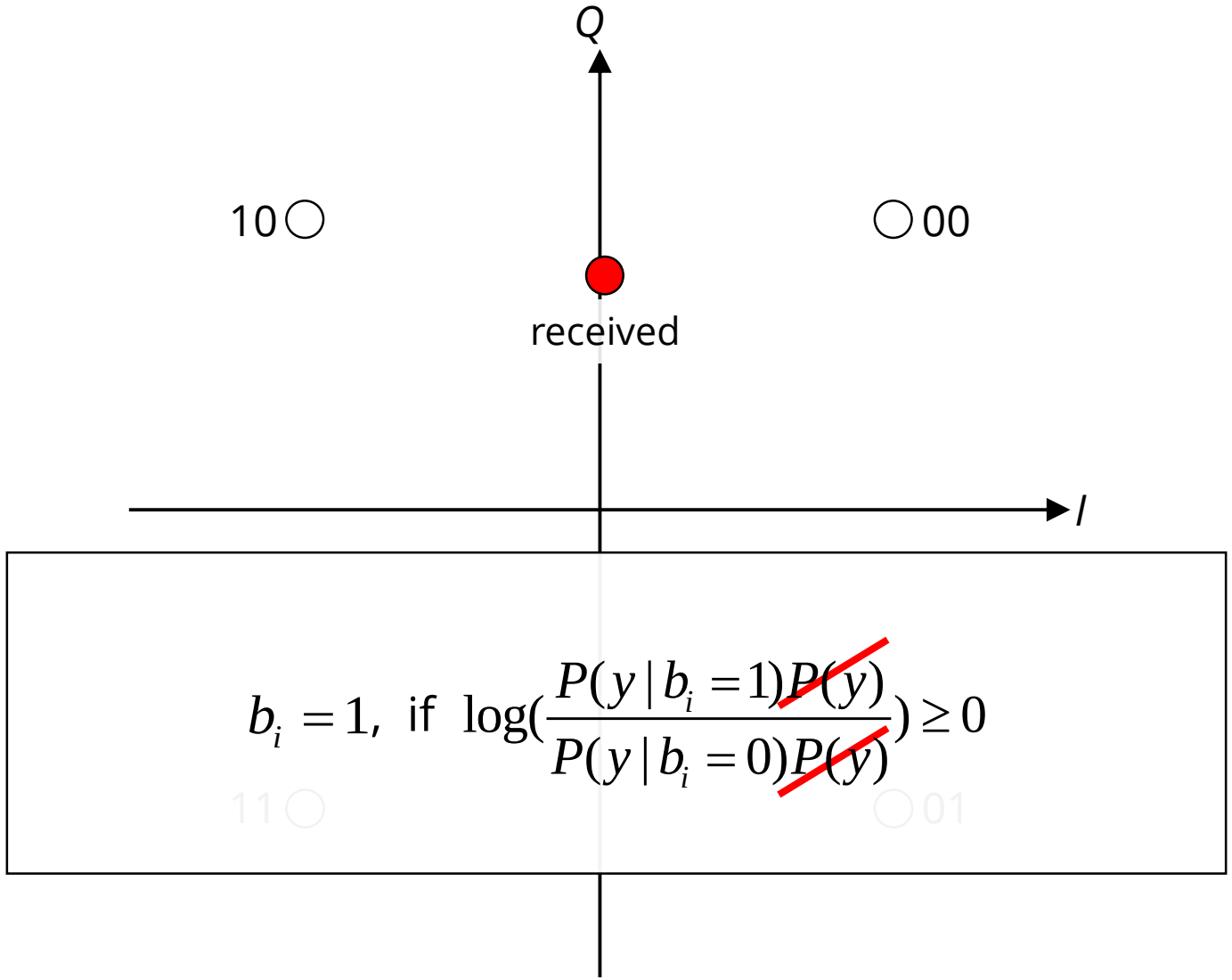


$$P(b_i|y) = \frac{P(y|b_i)P(b_i)}{P(y)} = \frac{P(y|b_i)}{P(y)}\frac{P(b_i)}{P(y)} \qquad \text{by Bayes' rule}$$

$$= \frac{P(y|b_i)}{P(y)} 2P(y) = P(y|b_i)P(y), \quad i = 1,2$$

Soft Bit-based Network Coding – What is Soft Bit?

Case 2



Soft Bit-based Network Coding – What is Soft Bit?

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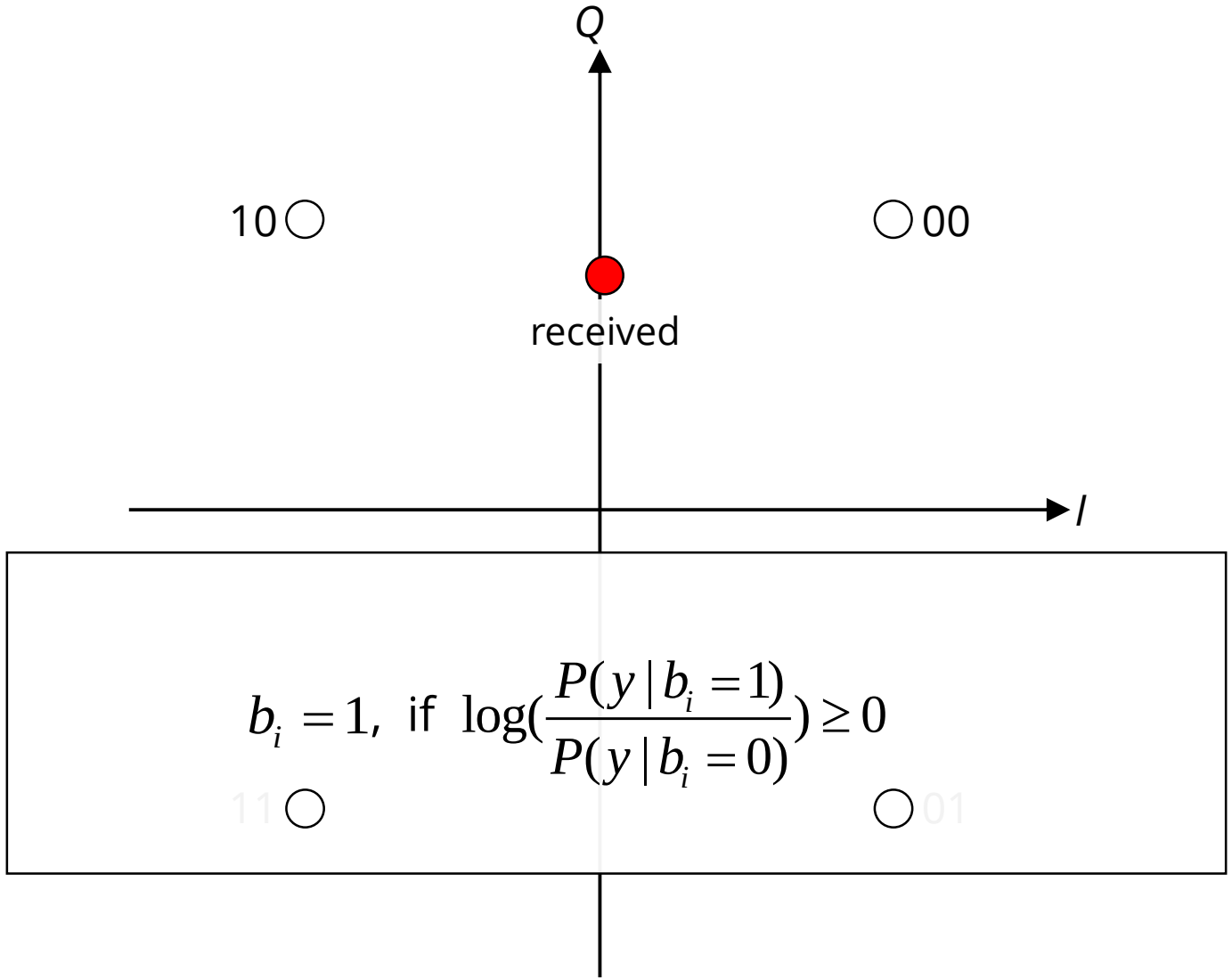

Soft Bit-based Network Coding – Encoding and Decoding

- **Modulation Mapping Rule**
  - ✓ Binary $0 \rightarrow +1$ and Binary $1 \rightarrow -1$
  - ✓ $b_1 \oplus b_2 \triangleq f(b_1)f(b_2)$
  - ✓ ex) $0 \oplus 1 = 1$
  - ✓ ex) $+1 \times -1 = -1 \rightarrow 1$

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- **Therefore,**
  - ✓ $P(b_1 \oplus b_2 = +1) = P(b_1 = +1)P(b_2 = +1) + P(b_1 = -1)P(b_2 = -1)$
  - ✓ $P(b_1 \oplus b_2 = -1) = P(b_1 = +1)P(b_2 = -1) + P(b_1 = -1)P(b_2 = +1)$
  - ✓ where $P(b = +1) = \frac{e^L}{1+e^L}$ and $P(b = -1) = \frac{1}{1+e^L}$

J. Hagenauer, E. Offer, and L. Papke, "Iterative Decoding of Binary Block and Convolutional Codes,"*IEEE Transactions On Information Theory*, Mar. 1996.

Soft Bit-based Network Coding – Encoding and Decoding

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  - ✓ Binary $0 \rightarrow +1$ and Binary $1 \rightarrow -1$
  - ✓ $b_1 \oplus b_2 \triangleq f(b_1)f(b_2)$
  - ✓ ex) $0 \oplus 1 = 1$
  - ✓ ex) $+1 \times -1 = -1 \rightarrow 1$

$$L(b_1 \oplus b_2) = \log\left(\frac{P(y|b_1 \oplus b_2 = +1)}{P(y|b_1 \oplus b_2 = -1)}\right) = \frac{1 + e^{L(b_1)+L(b_2)}}{e^{L(b_1)} + e^{L(b_2)}}$$
$$\approx sign(L(b_1)) \cdot sign(L(b_2)) \cdot \min(|L(b_1)|, |L(b_2)|)$$

- **Therefore,**
  - ✓ $P(b_1 \oplus b_2 = +1) = P(b_1 = +1)P(b_2 = +1) + P(b_1 = -1)P(b_2 = -1)$
  - ✓ $P(b_1 \oplus b_2 = -1) = P(b_1 = +1)P(b_2 = -1) + P(b_1 = -1)P(b_2 = +1)$
  - ✓ where $P(b = +1) = \frac{e^L}{1+e^L}$ and $P(b = -1) = \frac{1}{1+e^L}$

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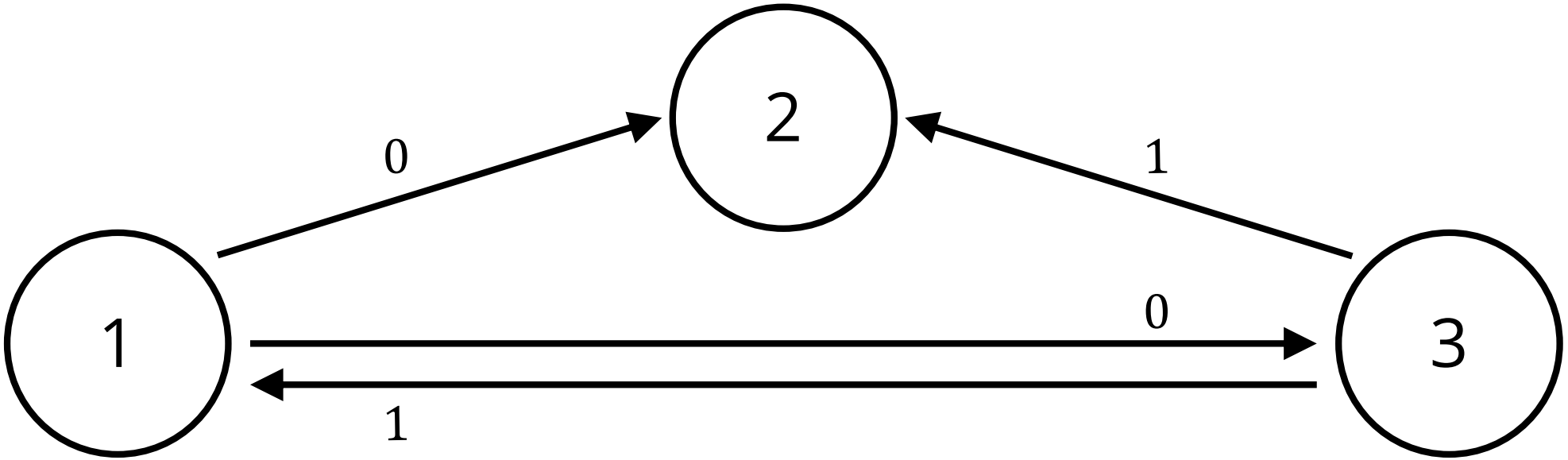
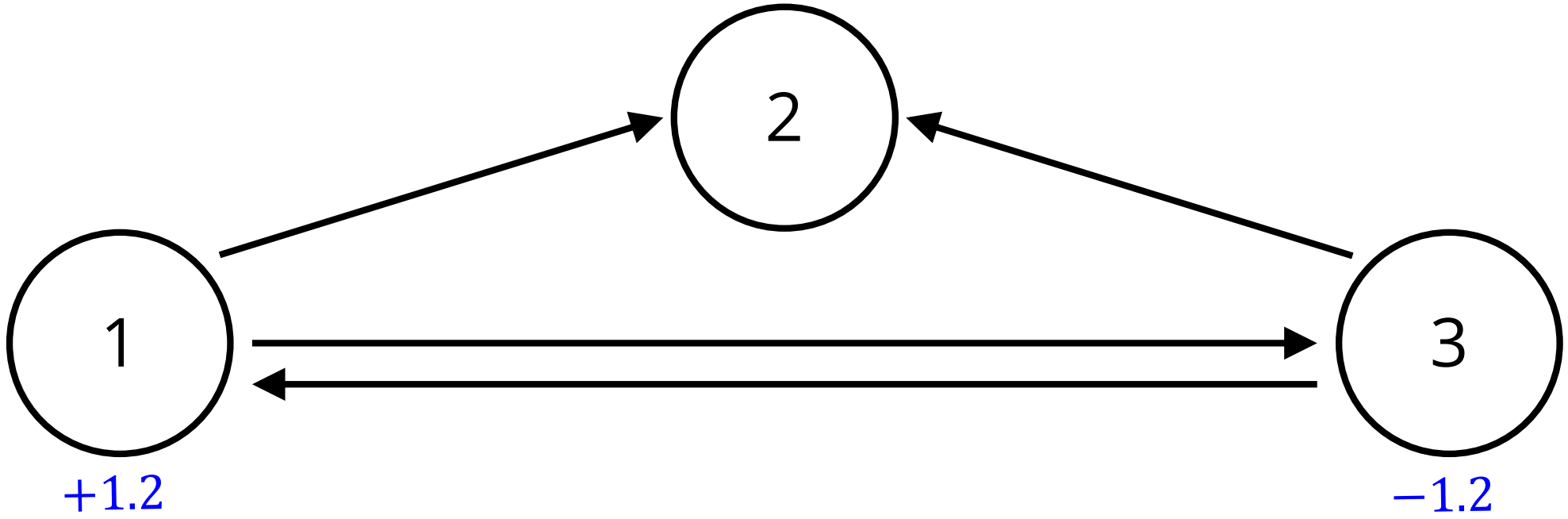
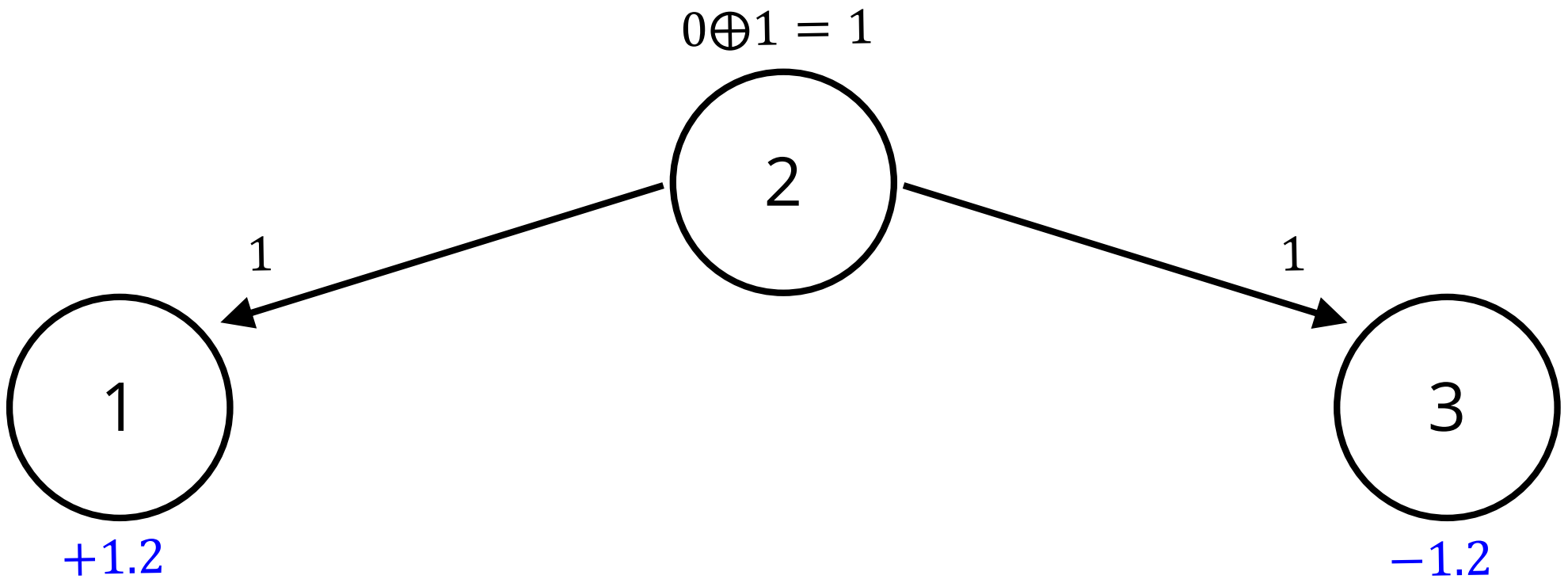
Soft Bit-based Network Coding – Example (1st Time Slot)



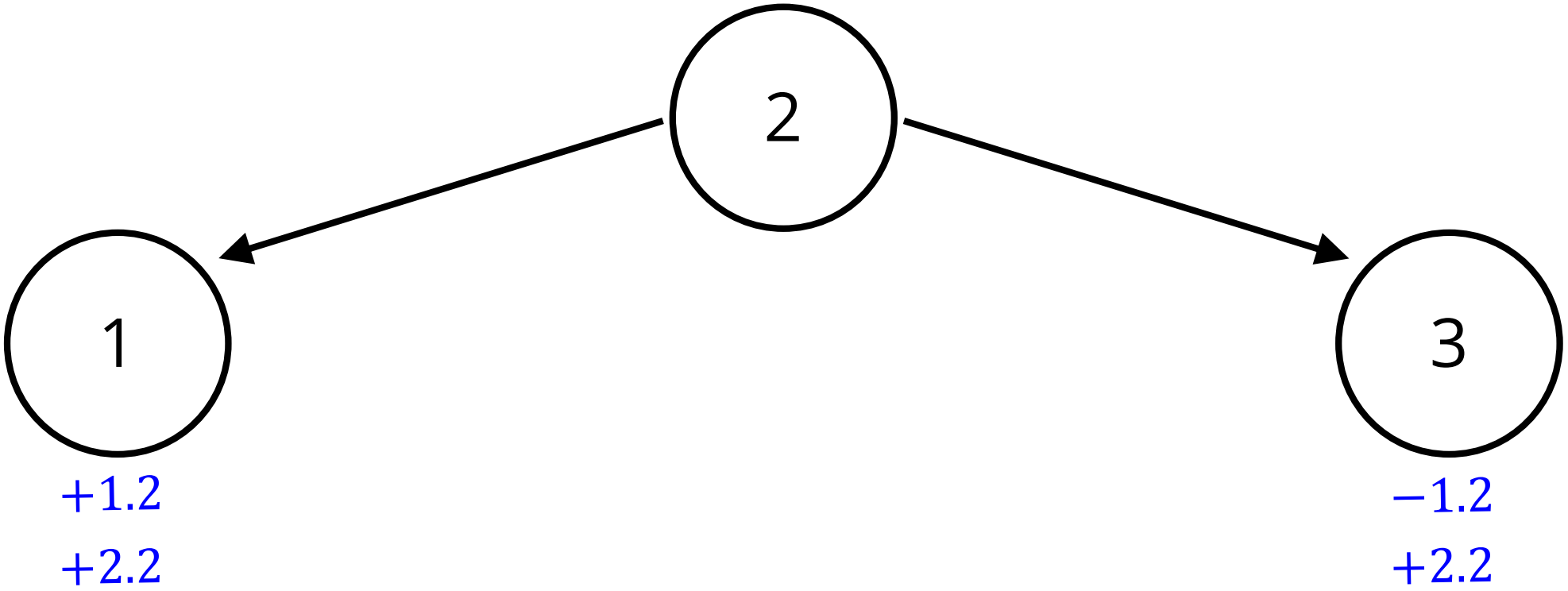
Soft Bit-based Network Coding – Example (1st Time Slot)



Soft Bit-based Network Coding – Example (2nd Time Slot)

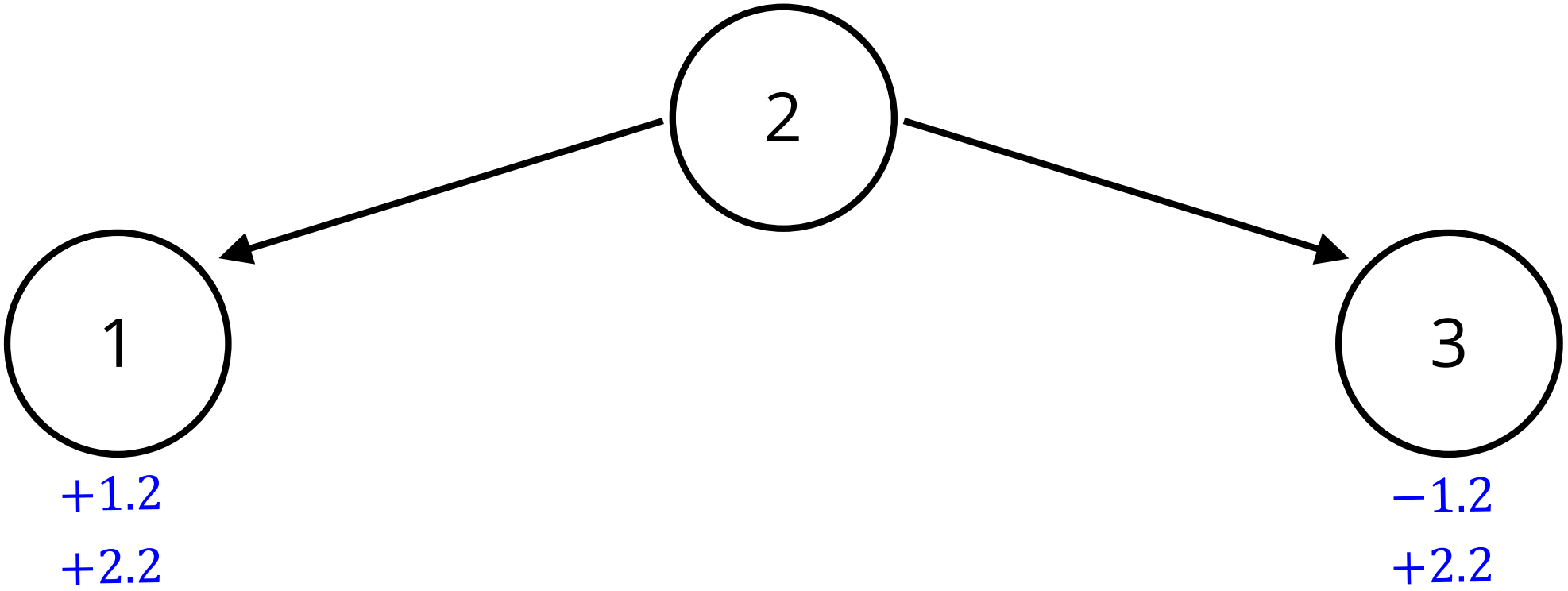


Soft Bit-based Network Coding – Example (2nd Time Slot)



Soft Bit-based Network Coding – Example (2nd Time Slot)

$$L(b_1 \oplus b_2) \approx sign(-1.2)) \cdot sign(+2.2) \cdot \min(|-1.2)|, |+2.2|)$$
$$\approx -1 \cdot +1 \cdot 1.2$$
$$\approx -1.2$$



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$$L(b_1 \oplus b_2) \approx sign(-1.2)) \cdot sign(+2.2) \cdot \min(|-0.2)|, |+2.2|)$$
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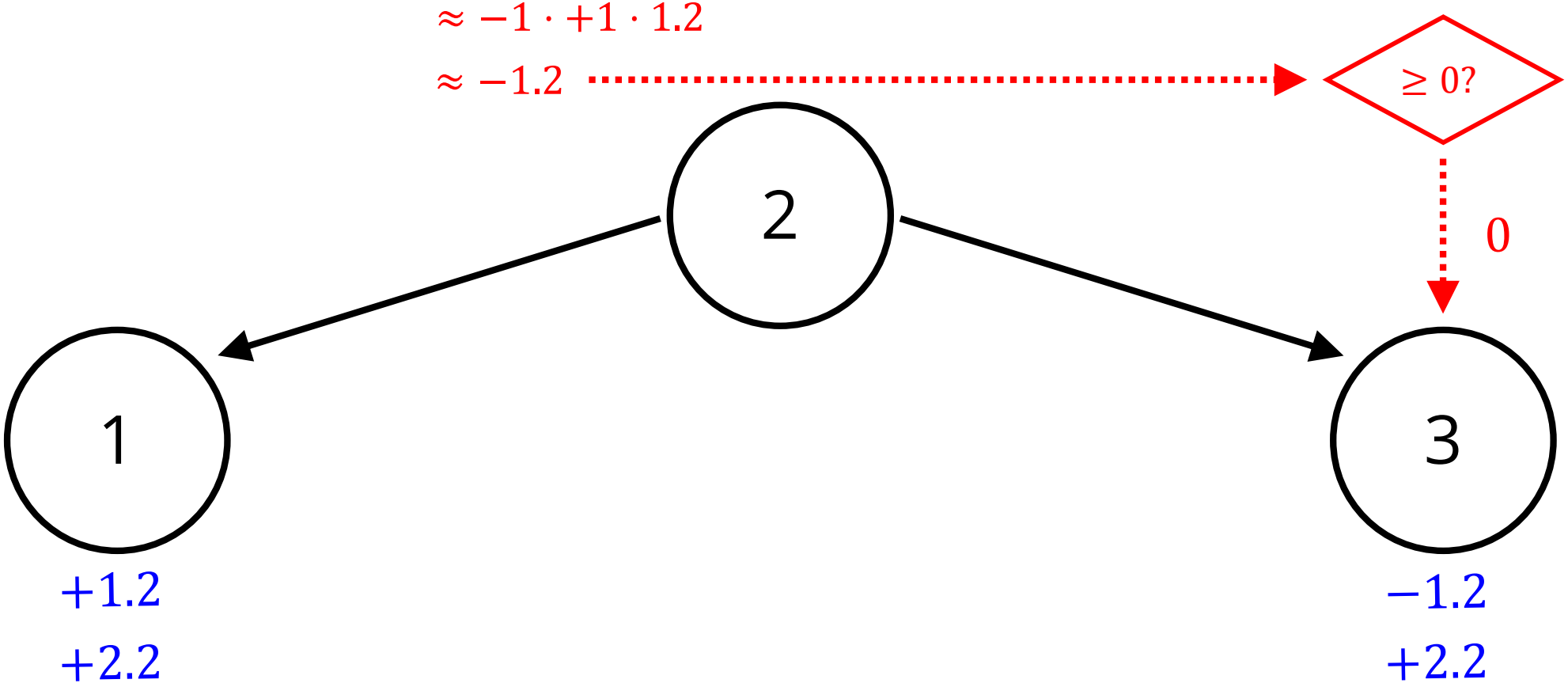
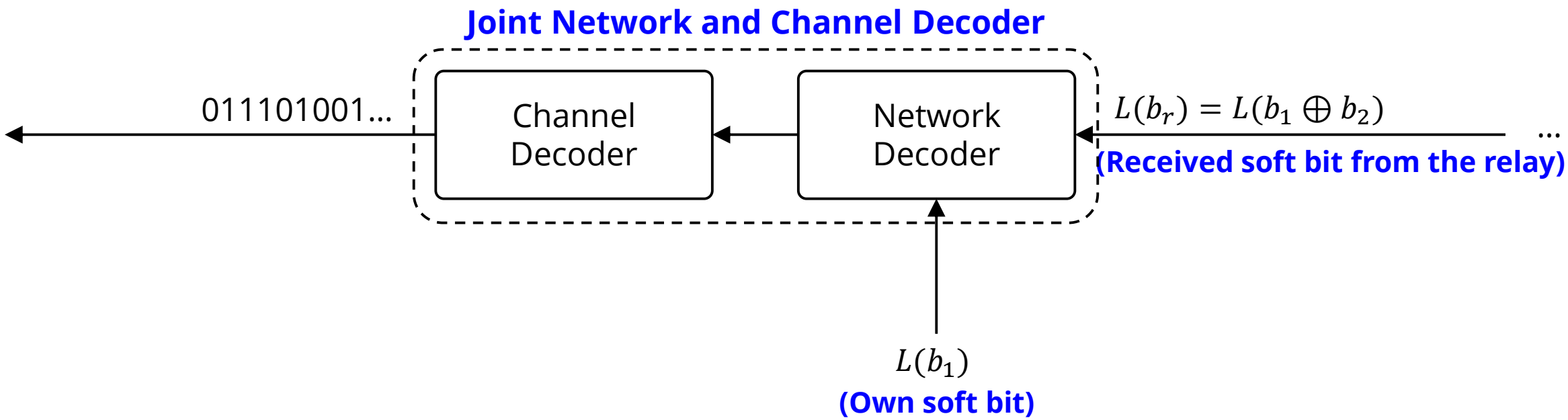
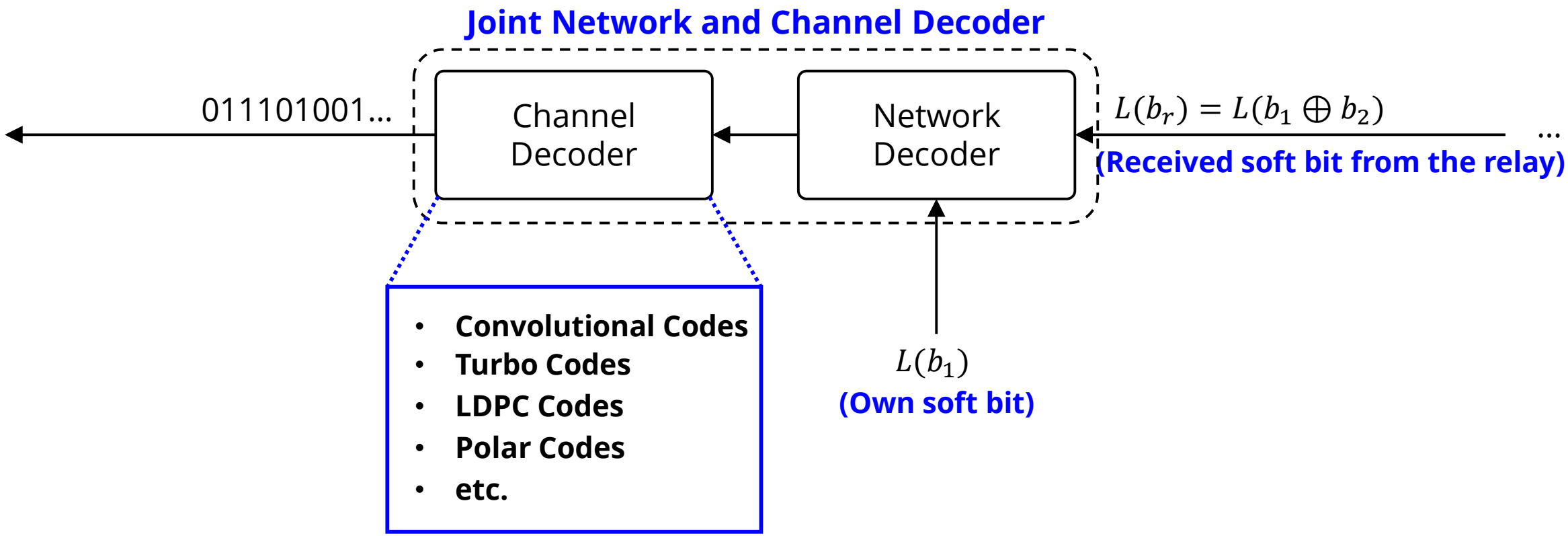


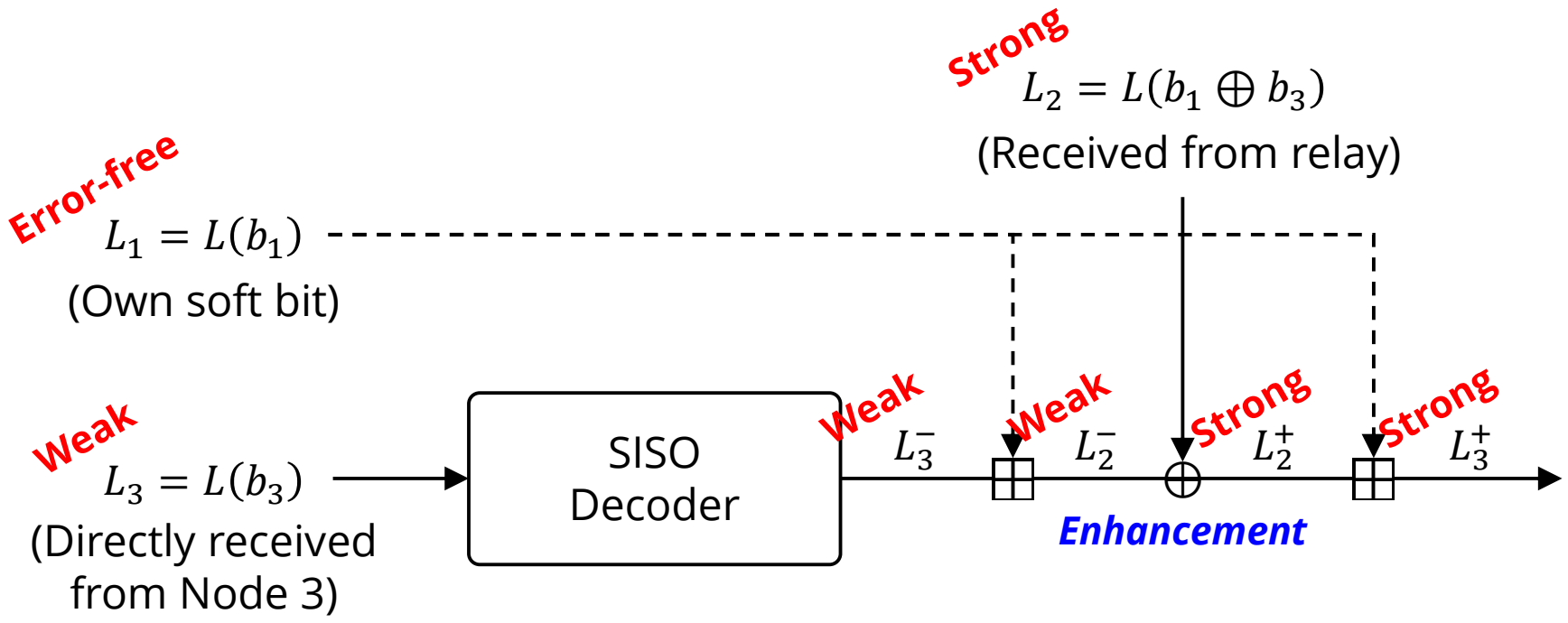
Soft Bit-based Network Coding – Decoding



Soft Bit-based Network Coding – Decoding



Soft Bit-based Network Coding – Decoding (SISO Decoder Example)



※ SISO: Soft Input Soft Output

Summary

- **Motivations**
- **Physical-layer Network Coding (PNC)**
- **Analog Network Coding (ANC)**
- **Soft Bit-based Network Coding**